Limits

Suppose that f is a function defined on some open interval containing the number a. The function f may or may not be defined at a.

Limit notation $\lim_{x \to a} f(x) = L$ is read "the limit of $f(x)$ as x approaches a equals the number L." This means as x gets closer to a, but remains unequal to a, the corresponding values of $f(x)$ get closer to L.

Finding a Limit Using a Table: Construct a table to find the indicated limit.

1) $\lim_{x \to 4} 3x^2$ = 48

3.9	3.99	3.999	4	4.01	4.1
45.63	47.76	47.976	48	48.12	50.43

2) $\lim_{x \to 0} \dfrac{x+1}{x^2+1}$ = 1

-0.9	-0.01	-0.001	0	0.001	0.01
0.89	0.98	0.999	1	1.001	1.01099

3) $\lim_{x \to 0} \dfrac{\sin x}{x}$ = 1

-0.1	-0.01	-0.001	0	0.001	0.01
0.998	0.999	0.999	DNE	0.999	0.999

4) $\lim_{x \to 0} \dfrac{\cos x - 1}{x}$ = 0

-0.1	-0.01	-0.001	0	0.001	0.01
$1.5 \cdot 10^6$	$1.5 \cdot 10^6$	$1.5 \cdot 10^{-7}$	DNE	$\frac{1}{2} \cdot 10^{-7}$	$-2 \cdot 10^{-6}$

Finding a Limit Using a graph: Use the graph of f to find the indicated limit and function value.
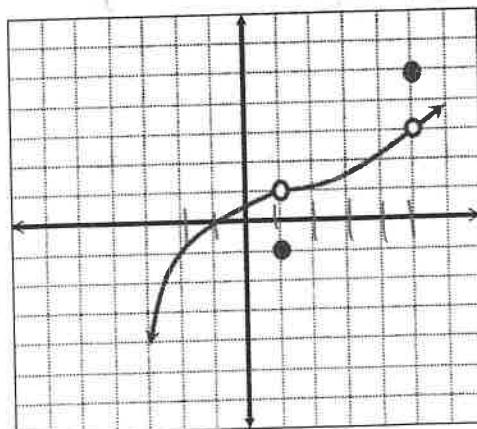
5) $\lim_{x \to 1} f(x)$ 1

6) $f(1)$ -1

7) $\lim_{x \to -2} f(x)$ -1

8) $f(-2) = -1$

9) $\lim_{x \to 5} f(x)$ 3

10) $f(5)$ 5

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Equal and Unequal One-Sided Limits

Equal One-Sided Limits:

$\lim\limits_{x \to a} f(x) = L$ if and only if both $\lim\limits_{x \to a^-} f(x) = L$ and $\lim\limits_{x \to a^+} f(x) = L$.

Equal One-Sided Limits:

If $\lim\limits_{x \to a^-} f(x) = L$ and $\lim\limits_{x \to a^+} f(x) = M$, where $L \neq M$, then $\lim\limits_{x \to a} f(x)$ does not exist.

One-Side Limits: The graph of a function f is given. Use the graph to find the indicated limits and function values, or state that a limit or function value does not exist.
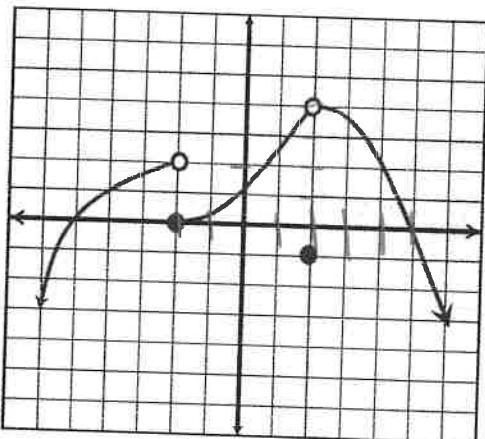
11) $\lim\limits_{x \to -2^-} f(x)$

2

12) $\lim\limits_{x \to -2^+} f(x)$

0

13) $\lim\limits_{x \to -2} f(x)$

DNE

14) $f(-2)$

0

15) $\lim\limits_{x \to 2^-} f(x)$

4

16) $\lim\limits_{x \to 2^+} f(x)$

4

17) $\lim\limits_{x \to 2} f(x)$

4

18) $f(2)$

-1

19) $\lim\limits_{x \to 5^-} f(x)$

0

20) $\lim\limits_{x \to 5^+} f(x)$

0

21) $\lim\limits_{x \to 5} f(x)$ 0

22) $f(5)$ 0

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