

Lesson 5: The Matrix is All Around You

Do Now:

Multiply the matrices given, if possible.

$$\begin{matrix} 3 \times 2 & 2 \times 3 & & 3 \times 3 \\ \begin{bmatrix} -11 & 5 \\ 7 & 2 \\ 6 & 14 \end{bmatrix} & \begin{bmatrix} 4 & -3 & 2 \\ 5 & 2 & 1 \end{bmatrix} & = & \begin{bmatrix} -14 & 43 & -17 \\ 38 & -17 & 16 \\ 94 & 10 & 26 \end{bmatrix} \end{matrix}$$

Ex:  $-11(4) + 5(5) = -44 + 25 = -19$

$$\begin{matrix} 2 \times 2 & 2 \times 2 & & 2 \times 2 \\ \begin{bmatrix} 7 & -2 \\ -3 & 5 \end{bmatrix} & \begin{bmatrix} 2 & 2 \\ -5 & 6 \end{bmatrix} & = & \begin{bmatrix} 24 & 2 \\ -31 & 24 \end{bmatrix} \end{matrix}$$

Ex:  $7(2) + (-2)(-5) = 14 + 10 = 24$

Example 1

When dealing with matrices we name them with Capital letters.

Define a 2x2 matrix called matrix A.

$$A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Define another 2x2 matrix called matrix B.

$$B \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

We can create matrix equations like we do with "normal" numbers. Solve for matrix X in both matrix equations:

$XD = E$

$$\begin{aligned} X D D^{-1} &= E D^{-1} \\ X &= E D^{-1} \end{aligned}$$

$XD - E = F$

$$\begin{aligned} XD &= F + E \\ X D D^{-1} &= (F + E) D^{-1} \\ X &= (F + E) D^{-1} \end{aligned}$$

It's weird but we can't Divide matrices. Instead we must multiply by the Inverse.

It's like fractions! So how would those equations look?

$XD=E$

$XD-E=F$

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Because matrices contain LOTS of information, this makes doing certain tasks VERY easy.

**Example 2:**

Solve the system of equations.

$x + y + z = 6$

$2y + 5z = -4$

$2x + 5y - z = 27$

The Set Up

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

The Matrix Solution

$$\begin{aligned} AX &= B \\ \cancel{A}^{-1}AX &= \cancel{A}^{-1}B \\ X &= A^{-1}B \end{aligned}$$

$$\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \begin{matrix} x=5 \\ y=3 \\ z=-2 \end{matrix}$$

Calculator:  
 $2^{nd} \rightarrow X^{-1}$  (Matrix)  
 Edit: change Matrices  
 Names: Use them.

**Practice:**

Solve the system of equations using matrices.

$$3x + y = 3$$

$$4x - 3y = 17$$

$$\begin{matrix} A & & X & & B \\ \begin{bmatrix} 3 & 1 \\ 4 & -3 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} 3 \\ 17 \end{bmatrix} \end{matrix}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad \begin{matrix} x = 2 \\ y = -3 \end{matrix}$$

1)  $3x - y + 4z = -17$   
 $4x + 3y - 5z = 4$   
 $x + 6y + 2z = -6$

$$\begin{bmatrix} 3 & -1 & 4 \\ 4 & 3 & -5 \\ 1 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -17 \\ 4 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} -2.27317... \\ 0.209756... \\ -2.49268... \end{bmatrix}$$

$x \approx -2.27 \quad y \approx 0.21 \quad z \approx -2.49$

2)  $m + n + p + q = 7$   
 $-2m + 4n - p + 3q = 1$   
 $4m - 2n + 4p + q = 4$   
 $-m + 2n - 3p - 2q = 8$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 4 & -1 & 3 \\ 4 & -2 & 4 & 1 \\ -1 & 2 & -3 & -2 \end{bmatrix} \begin{bmatrix} m \\ n \\ p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} -3 \\ 9 \\ 11 \\ -10 \end{bmatrix} \quad \begin{matrix} m = -3 \\ n = 9 \\ p = 11 \\ q = -10 \end{matrix}$$

3)  $s + t - u = 5$   
 $2s - 5t + 3u = 10$   
 $-s + 6t - 7u = 2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -5 & 3 \\ -1 & 6 & -7 \end{bmatrix} \begin{bmatrix} s \\ t \\ u \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 13/3 \\ -5/3 \\ -7/3 \end{bmatrix} \quad \begin{matrix} s = 13/3 \\ t = -5/3 \\ u = -7/3 \end{matrix}$$

4)  $a + 2b + 3c + 4d + 5e = 6$   
 $-a - 3b - 2c - 5d - 4e = 12$   
 $4a + 7b - 7c + 8d - e = -2$   
 $-3a + 2b + 8c - 2e = 14$   
 $6a - 5b - 2c + d - 4e = 0$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & -3 & -2 & -5 & -4 \\ 4 & 7 & -7 & 8 & -1 \\ -3 & 2 & 8 & 0 & -2 \\ 6 & -5 & -2 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ -2 \\ 14 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 30568 \\ 24428 \\ 7995 \\ -29402 \\ 3969 \end{bmatrix}$$

$$\begin{matrix} a = \frac{30568}{941} & b = \frac{24428}{941} & c = \frac{7995}{941} \\ d = \frac{-29402}{941} & e = \frac{3969}{941} \end{matrix}$$

$$\begin{aligned} 5) \quad & 2h - j + 4k - 2m = 23 \\ & 4h + 2j - k + 3m = -1 \\ & h - 5j + 8k - 4m = 19 \\ & -3h + j - 2k = -6 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 4 & -2 \\ 4 & 2 & -1 & 3 \\ 1 & -5 & 8 & -4 \\ -3 & 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} h \\ j \\ k \\ m \end{bmatrix} = \begin{bmatrix} 23 \\ -1 \\ 19 \\ -6 \end{bmatrix}$$

$$h = \frac{17}{7} \quad j = \frac{46}{7} \quad k = \frac{37}{14} \quad m = -\frac{99}{14}$$

What if you don't have my graphing calculators with you?

[matrixcalc.org/en/](http://matrixcalc.org/en/)

$$-3x - 2y + 4z = 9$$

$$3y - 2z = 5$$

$$4x - 3y + 2z = 7$$

$$\begin{bmatrix} -3 & -2 & 4 \\ 0 & 3 & -2 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

$$x = 3$$

$$y = 7$$

$$z = 8$$

$$2x - 4y + z = 3$$

$$x - 3y + z = 5$$

$$3x - 7y + 2z = 12$$

$$\begin{bmatrix} 2 & -4 & 1 \\ 1 & -3 & 1 \\ 3 & -7 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

↓  
Not invertible!  
No solution!

Homework: Use the online calculator

Solve each system using a matrix equation.

$$\begin{aligned} 35. \quad x - 2y &= 8 \\ 3x + 4y &= 4 \end{aligned}$$

$$x = 4 \quad y = -2$$

$$\begin{aligned} 37. \quad 2x - 3y &= -8 \\ 4x + y &= -2 \end{aligned}$$

$$x = -1 \quad y = 2$$

$$\begin{aligned} 39. \quad x + y + z &= 0 \\ 2x - y + z &= 1 \\ x + y - 2z &= 2 \end{aligned}$$

$$x = \frac{7}{9} \quad y = -\frac{1}{9} \quad z = \frac{-2}{3}$$

$$\begin{aligned} 41. \quad 2x + 2y + z &= 9 \\ x + z &= 4 \\ 4y - 3z &= 17 \end{aligned}$$

$$x = 14 \quad y = -7 \quad z = -15$$

$$\begin{aligned} 36. \quad 3x + y &= 1 \\ -7x - 2y &= -1 \end{aligned}$$

$$x = -1 \quad y = 4$$

$$\begin{aligned} 38. \quad 5x + 3y &= 9 \\ -2x + y &= -8 \end{aligned}$$

$$x = 3 \quad y = -2$$

$$\begin{aligned} 40. \quad 2x + y - 2z &= 4 \\ x + 3y - z &= -3 \\ 3x + 4y - z &= 7 \end{aligned}$$

$$x = 6 \quad y = -2 \quad z = 3$$

$$\begin{aligned} 42. \quad 2x + 3y - 2z &= 10 \\ 3x - 2y + 2z &= 0 \\ 4x - y + 3z &= -1 \end{aligned}$$

$$x = 2 \quad y = 0 \quad z = -3$$

