

Answers 4.11

Problem Set Sample Solutions

1. For each of the following, write the inverse of the function given.

a. $f = \{(1, 3), (2, 15), (3, 8), (4, -2), (5, 0)\}$

$f^{-1} = \{(3, 1), (15, 2), (8, 3), (-2, 4), (0, 5)\}$

b. $g = \{(0, 5), (2, 10), (4, 15), (6, 20)\}$

$g^{-1} = \{(5, 0), (10, 2), (15, 4), (20, 6)\}$

c. $h = \{(1, 5), (2, 25), (3, 125), (4, 625)\}$

$h^{-1} = \{(5, 1), (25, 2), (125, 3), (625, 4)\}$

d.

x	1	2	3	4
$f(x)$	3	12	27	48

x	3	12	27	48
$f^{-1}(x)$	1	2	3	4

e.

x	-1	0	1	2
$g(x)$	3	6	12	24

x	3	6	12	24
$g^{-1}(x)$	-1	0	1	2

f.

x	1	10	100	1,000
$h(x)$	0	1	2	3

x	0	1	2	3
$h^{-1}(x)$	1	10	100	1,000

g. $y = 2x$

$y = \frac{1}{2}x$

h. $y = \frac{1}{3}x$

$y = 3x$

i. $y = x - 3$

$y = x + 3$

j. $y = -\frac{2}{3}x + 5$

$$y = -\frac{3}{2}x + \frac{15}{2}$$

k. $2x - 5y = 1$

$$2y - 5x = 1$$

l. $-3x + 7y = 14$

$$-3y + 7x = 14$$

m. $y = \frac{1}{3}(x - 9)^3$

$$y = \sqrt[3]{3x + 9}$$

n. $y = \frac{5}{3x - 4}, x \neq \frac{4}{3}$

$$y = \frac{5}{3x} + \frac{4}{3}$$

o. $y = 2x^7 + 1$

$$y = \sqrt{\frac{1}{2}x - \frac{1}{2}}$$

p. $y = \sqrt[3]{x}$

$$y = x^5$$

q. $y = \frac{x + 1}{x - 1}, x \neq 1$

$$y = \frac{x + 1}{x - 1}$$

2. For each part in Problem 1, state the domain, D , and range, R , of the inverse function.

a. $D = \{-2, 0, 3, 8, 15\}$
 $R = \{0, 1, 2, 3, 4, 5\}$

b. $D = \{5, 10, 15, 20\}$
 $R = \{0, 2, 4, 6\}$

c. $D = \{5, 25, 125, 625\}$
 $R = \{1, 2, 3, 4\}$

d. $D = \{3, 12, 27, 48\}$
 $R = \{1, 2, 3, 4\}$

e. $D = \{3, 6, 12, 24\}$
 $R = \{-1, 0, 1, 2\}$

f. $D = \{0, 1, 2, 3\}$
 $R = \{1, 10, 100, 1000\}$

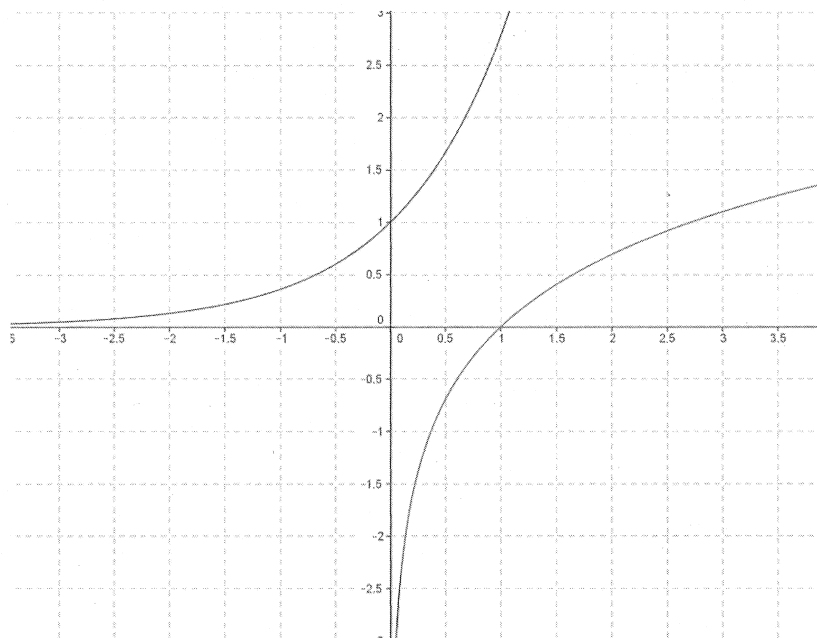
g. Both domain and range are all real numbers.

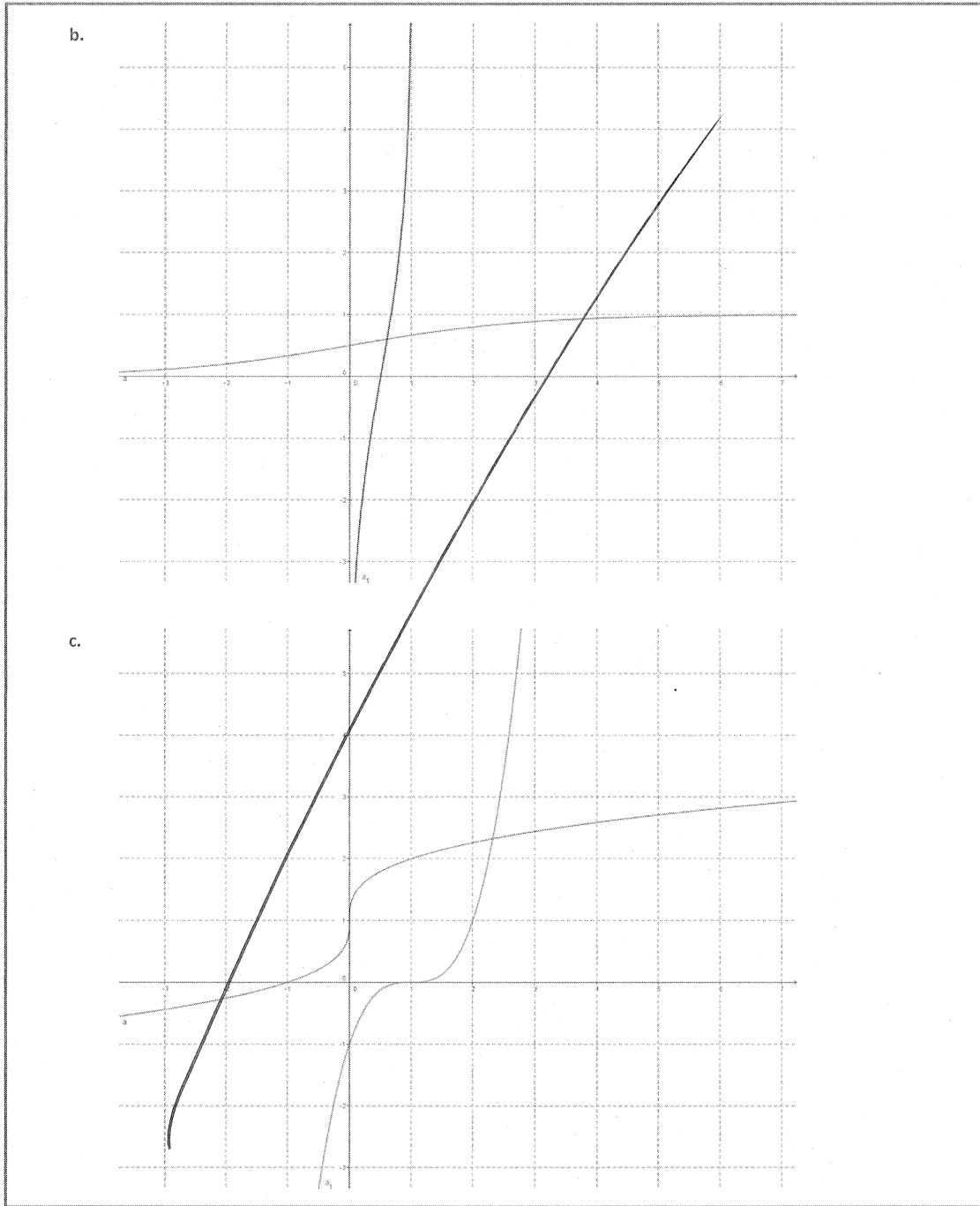
h. Both domain and range are all real numbers.

- i. Both domain and range are all real numbers.
- j. Both domain and range are all real numbers.
- k. Both domain and range are all real numbers.
- l. Both domain and range are all real numbers.
- m. Both domain and range are all real numbers.
- n. The domain is all real numbers except $x = 0$, and the range is all real numbers except $y = \frac{4}{3}$.
- o. Both domain and range are all real numbers.
- p. Both domain and range are all real numbers.
- q. Both domain and range are all real numbers except 1.

3. Sketch the graph of the inverse function for each of the following functions:

a.





Lesson Summary

- **INVERTIBLE FUNCTION:** Let f be a function whose domain is the set X and whose image (range) is the set Y . Then, f is *invertible* if there exists a function g with domain Y and image (range) X such that f and g satisfy the property:

$$\text{For all } x \text{ in } X \text{ and } y \text{ in } Y, f(x) = y \text{ if and only if } g(y) = x.$$

The function g is called the *inverse* of f .

- If two functions whose domain and range are a subset of the real numbers are inverses, then their graphs are reflections of each other across the diagonal line given by $y = x$ in the Cartesian plane.
- If f and g are inverses of each other, then:
 - The domain of f is the same set as the range of g .
 - The range of f is the same set as the domain of g .
- The inverse of a function f is denoted f^{-1} .
- In general, to find the formula for an inverse function g of a given function f :
 - Write $y = f(x)$ using the formula for f .
 - Interchange the symbols x and y to get $x = f(y)$.
 - Solve the equation for y to write y as an expression in x .
 - Then, the formula for f^{-1} is the expression in x found in the previous step.

Problem Set

3. For each of the following, write the inverse of the function given.

a. $f = \{(1, 7), (-22, 15), (3, 8), (4, -2), (5, 0)\}$

b. $g = \{(0, 5), (2, 10), (4, 15), (6, 20)\}$

c. $h = \{(1, 5), (2, 25), (3, 125), (4, 625)\}$

d.

x	1	2	9	24
$f(x)$	0	12	27	4

e.

x	-1	0	1	2
$g(x)$	3	6	12	24

f.

x	1	10	100	1,000
$h(x)$	0	1	2	3

g. $y = 2x$

h. $y = \frac{1}{3}x$

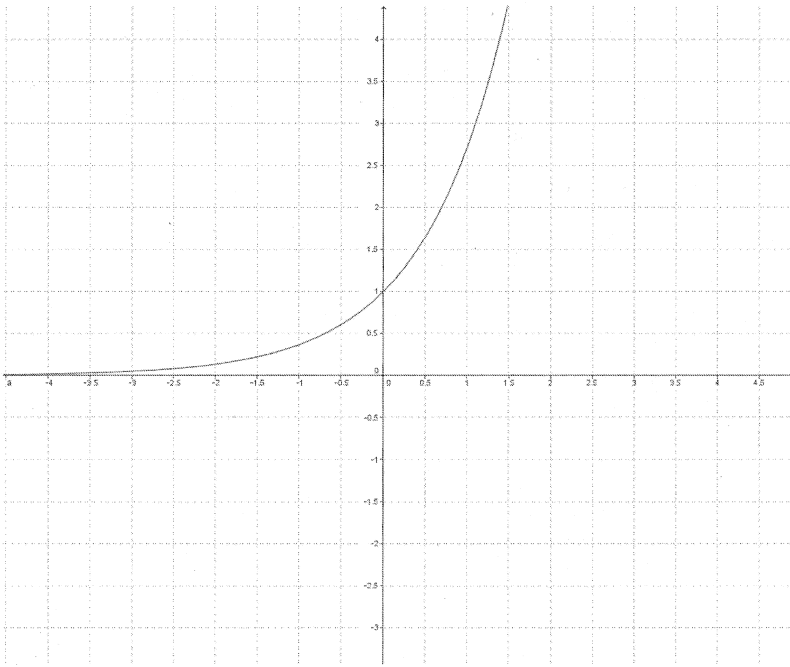
i. $y = x + 6$

j. $y = -\frac{2}{9}x + 7$

- k. $3x - 5y = 2$
- l. $-3x + 7y = 14$
- m. $y = \frac{1}{3}(x - 9)^3$

4. Sketch the graph of the inverse function for each of the following functions:

a.



Find the inverse of each function.

9) $h(x) = \sqrt[3]{x} - 3$
 $y = \sqrt[3]{x} - 3$
 $x = \sqrt[3]{y} - 3$
 $x + 3 = \sqrt[3]{y}$
 $(x + 3)^3 = y$

10) $g(x) = \frac{1}{x} - 2$
 $y = \frac{1}{x} - 2$
 $x = \frac{1}{y} - 2$
 $x + 2 = \frac{1}{y}$
 $y(x + 2) = 1$
 $y = \frac{1}{x + 2}$

11) $h(x) = 2x^3 + 3$
 $y = 2x^3 + 3$
 $x = \sqrt[3]{\frac{y - 3}{2}}$
 $x - 3 = 2y^3$
 $\frac{x - 3}{2} = y^3$
 $\sqrt[3]{\frac{x - 3}{2}} = y$

12) $g(x) = -4x + 1$
 $y = -4x + 1$
 $x = -\frac{y - 1}{4}$
 $x - 1 = -4y$
 $\frac{x - 1}{-4} = y$

Challenge! Find the inverse!

① If $f(x) = \frac{4x-3}{2x+1}$, find $f^{-1}(x)$.

② If $f(x) = \frac{5}{6}x - \frac{3}{4}$, find $f^{-1}(x)$.

③ If $f(x) = -(x+2)^2 - 1$, find $f^{-1}(x)$.

④ If $f(x) = -3x + 11$, find $f^{-1}(x)$.

⑤ If $f(x) = \sqrt[5]{x+2} - 3$, find $f^{-1}(x)$.

⑥ If $f(x) = \frac{2x-5}{3}$, find $f^{-1}(x)$.

① $y = \frac{4x-3}{2x+1}$

$x = \frac{4y-3}{2y+1}$

$(2y+1)x = 4y-3$

$2yx + x = 4y-3$

$x+3 = 4y-2yx$

$x+3 = y(4-2x)$

$\frac{x+3}{4-2x} = y$

② $y = \frac{5}{6}x - \frac{3}{4}$

$x = \frac{5}{6}y - \frac{3}{4}$

$x + \frac{3}{4} = \frac{5}{6}y$

$\frac{6}{5}\left(x + \frac{3}{4}\right) = y$

③ $y = -(x+2)^2 - 1$

$x = -(y+2)^2 - 1$

$x+1 = -(y+2)^2$

$-(x+1) = (y+2)^2$

$\sqrt{-(x+1)} = y+2$

$\sqrt{-(x+1)} - 2 = y$

④

$y = -3x + 11$

$x = -3y + 11$

$x - 11 = -3y$

$\frac{x-11}{-3} = y$

⑤ $y = \sqrt[5]{x+2} - 3$

$x+3 = \sqrt[5]{x+2}$

$(x+3)^5 = x+2$

$(x+3)^5 - 2 = y$

⑥

$y = \frac{2x-5}{3}$

$x = \frac{2y-5}{3}$

$3x = 2y-5$

$3x+5 = 2y$

$\frac{3x+5}{2} = y$

