Lesson 13: Inverses of Logarithmic and Exponential Functions

(Test Wednesday)

Classwork

Opening Exercise

1. If f(x)=x³+1, find f⁻¹(x).

   $y = x^3+1$
   $x = y^3+1$
   $f^{-1}(x) = \sqrt[3]{x-1}$

2. Find f(2).

   $f(2) = 2^3+1 = 9$

3. Take your answer from part 2) and make it the input of f⁻¹(x). What do you get?

   $f^{-1}(9) = \sqrt[3]{9-1} = 2$

   $2 \rightarrow 9 \rightarrow 2$

4. Determine algebraically if $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$ are inverses.

   $f(g(x)) = \frac{(3x-2)+2}{3} = \frac{3x}{3} = x$

   $g(f(x)) = 3\left(\frac{x+2}{3}\right) - 2 = x+2-2 = x$

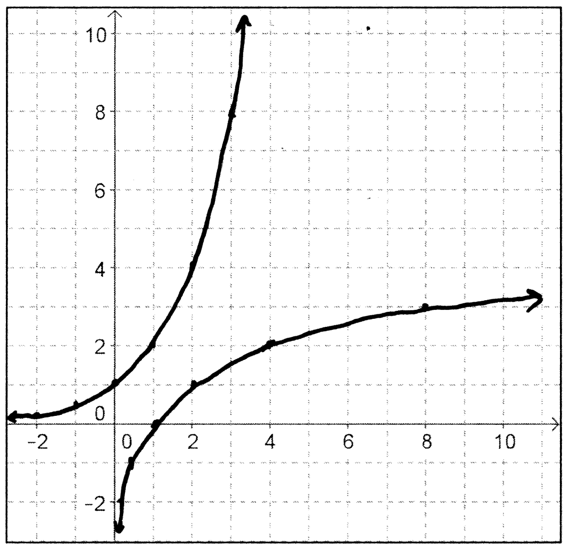
5. Determine algebraically if $f(x) = 3x^2 - 2$ and $g(x) = 3\sqrt{x+2}$ are inverses.

   $f(g(x)) = 3(3\sqrt{x+2})^2 - 2 = 27(x+2) - 2 = 27x+52$

   not inverses

Let $f(x) = 2^x$.

a. Complete the table, and use the points $(x, f(x))$ to create a sketch of the graph of $y = f(x)$.



x	$f(x)$
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

b. Create a table of values for the function f^{-1}, and sketch the graph of $y = f^{-1}(x)$ on the grid above.

x	f^{-1}
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

c. What type of function is f^{-1}? Explain how you know.

f^{-1} is logarithmic; f^{-1} tells us what power.

Example

Given $f(x) = 2^x$, use the definition of the inverse of a function and the definition of a logarithm to write a formula for $f^{-1}(x)$.

$y=2^x$

$x=2^y$

$\log_2(x)=y$

Exercises

1. Find the value of y in each equation. Explain how you determined the value of y.

   a. $y = \log_2(2^2)$

   $y=2$

   b. $y = \log_2(2^5)$

   $y=5$

   c. $y = \log_2(2^{-1})$

   $x=-1$

   d. $y = \log_2(2^x)$

   $y=x$

2. Let $f(x) = \log_2(x)$ and $g(x) = 2^x$.
   a. What is $f(g(x))$?

      $f(2^x) = \log_2(2^x) = x$

   b. Based on the results of part (a), what can you conclude about the functions f and g?

      f and g would probably be functions.

3. Find the value of y in each equation. Explain how you determined the value of y.
   a. $y = 3^{\log_3(3)}$

      $y = 3$

   b. $y = 3^{\log_3(9)}$

      $y = 9$

   c. $y = 3^{\log_3(81)}$

      $y = 81$

   d. $y = 3^{\log_3(x)}$

      $y = x$

4. Let $f(x) = \log_3(x)$ and $g(x) = 3^x$.
   a. What is $g(f(x))$?

   $g(\log_3(x)) = 3^{\log_3(x)} = x$

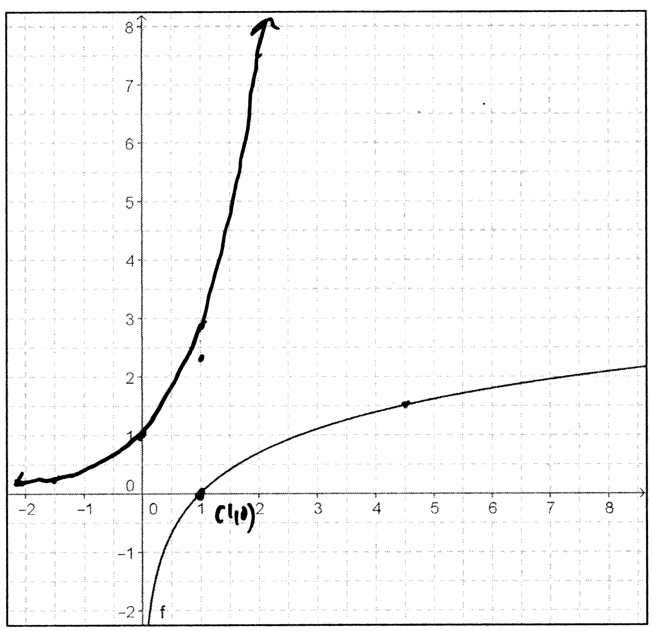
   b. Based on the results in part (a), what can you conclude about the functions f and g?

   f and g are probably inverses.

5. Verify by composition that the functions $f(x) = b^x$ and $g(x) = \log_b(x)$ for $b > 0$ are inverses of one another

   $f(g(x)) = f(\log_b(x)) = b^{\log_b(x)} = x$
   $g(f(x)) = g(b^x) = \log_b(b^x) = x$

6. The graph of $y = f(x)$, a logarithmic function, is shown below.



   a. Construct the graph of $y = f^{-1}(x)$.

   b. Estimate the base b of these functions. Explain how you got your answer.

   $b \approx 2.75$ b/c $\log_b(b) = 1$

   $b = 2.75$

7. Find the inverse of each function.

a. $f(x) = 2^{x-3}$

$y = 2^{x-3}$

$x = 2^{y-3}$

$\log_2(x) + 3 = f^{-1}(x)$

b. $g(x) = 2\log(x-1)$

$x = 2\log(y-1)$

$\frac{x}{2} = \log(y-1)$

$10^{x/2} = y-1$

$10^{x/2} + 1 = f^{-1}(x)$

c. $m(x) = \log_5(2x) - 3$

$x = \log_5(2y) - 3$

$x+3 = \log_5(2y)$

$5^{x+3} = 2y$

$f^{-1}(x) = \frac{5^{x+3}}{2}$

d. $h(x) = \ln(x) - \ln(x-1)$

$x = \ln(y) - \ln(y-1)$

$x = \ln\left(\frac{y}{y-1}\right)$

$e^x = \frac{y}{y-1}$

$e^x(y-1) = y$

$e^x y - y = e^x$

$y(e^x - 1) = e^x$

$h^{-1}(x) = \frac{e^x}{e^x - 1}$

e. $w(x) = 2 + e^{2x}$

$x = 2 + e^{2y}$

$x - 2 = e^{2y}$

$\ln(x-2) = 2y$

$w^{-1}(x) = \frac{\ln(x-2)}{2}$

f. $k(x) = 5 - 3^{-\frac{x}{2}}$

$x = 5 - 3^{-\frac{y}{2}}$

$x - 5 = -3^{-y/2}$

$-x + 5 = 3^{-y/2}$

$\log_3(-x+5) = -y/2$

$-2\log_3(-x+5) = k^{-1}(x)$

g. $k(x) = 5^{2x} - 4$

$x = 5^{2y} - 4$

$x + 4 = 5^{2y}$

$\log_5(x+4) = 2y$

$\frac{\log_5(x+4)}{2} = k^{-1}(x)$

Problem Set

1. Find the inverse of each function.

   a. $f(x) = 3^x$

   b. $f(x) = \left(\frac{1}{2}\right)^x$

   c. $g(x) = \ln(x-7)$

   d. $h(x) = \frac{\log_3(x+2)}{\log_3(5)}$

   e. $f(x) = 3(1.8)^{0.2x} + 3$

   f. $g(x) = \log_2\left(\sqrt[3]{x-4}\right)$

   g. $h(x) = \frac{5^x}{5^x+1}$

   h. $f(x) = 2^{-x+1}$

   i. $g(x) = \sqrt{\ln(3x)}$

   j. $h(x) = e^{\frac{1}{5}x+3} - 4$

a. $y=3^x$
$x=3^y$
$\log_3(x)=f^{-1}(x)$

b. $y=\left(\frac{1}{2}\right)^x$
$x=\left(\frac{1}{2}\right)^y$
$\log_{0.5}(x)=f^{-1}(x)$

c. $x=\ln(y-7)$
$e^x=y-7$
$e^x+7=g^{-1}(x)$

d. $y=\frac{\log_3(x+2)}{\log_3(5)}$
$x=\frac{\log_3(y+2)}{\log_3(5)}$
$\log_3(5)x=\log_3(y+2)$
$3^{\log_3(5)x}=y+2$
$3^{\log_3(5)x}-2=h^{-1}(x)$

e. $y=3(1.8)^{0.2x}+3$
$x=3(1.8)^{0.2y}+3$
$x-3=3(1.8)^{0.2y}$
$\frac{x-3}{3}=(1.8)^{0.2y}$
$\log_{1.8}\left(\frac{x-3}{3}\right)=0.2y$
$\frac{\log_{1.8}\left(\frac{x-3}{3}\right)}{0.2}=f^{-1}(x)$

f. $y=\log_2\left(\sqrt[3]{x-4}\right)$
$x=\log_2\left(\sqrt[3]{y-4}\right)$
$2^x=\sqrt[3]{y-4}$
$(2^x)^3=y-4$
$2^{3x}+4=g^{-1}(x)$

Answers, no work.

g. $h^{-1}(x)=\log_5\left(-\frac{x}{x-1}\right)$

h. $f^{-1}(x)=-\log_2(x)+1$

i. $g^{-1}(x)=\frac{1}{3}e^{x^2}$

j. $h^{-1}(x)=5\ln(x+4)-15$

+Khan Module: Solving Exponential Equations