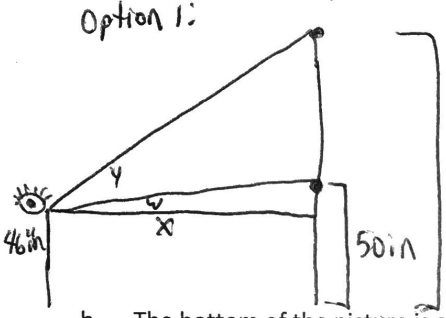
Problem Set

1. Consider the situation of sitting down with eye level at 46 in. Find the missing distances and heights for the following:

   a. The bottom of the picture is at 50 in. and the top is at 74 in. What is the optimal viewing distance?



$y = \tan^{-1}\left(\frac{74-46}{x}\right) - \tan^{-1}\left(\frac{50-46}{x}\right)$

Graph and find max!

$b = 74-46 = 28$

$a = 50-46 = 4$

OR: option 2:

$x = \sqrt{ab}$

$x = \sqrt{4 \cdot 28}$

$x = \sqrt{112}$

$x = 10.583$

a is distance from eye level to bottom

b is distance from eye level to top

$a = 4$

$b = 28$

   b. The bottom of the picture is at 52 in. and the top is at 60 in. What is the optimal viewing distance?

$b = 60-46 = 14$

$a = 52-46 = 6$

$x = \sqrt{14 \cdot 6}$

$x = \sqrt{84} = 9.165$

2. Ocean tides are an example of periodic behavior. At a particular harbor, data was collected over the course of 24 hours to create the following model: $y = 1.236 \sin\left(\frac{\pi}{3}x\right) + 1.798$, which gives the water level, y, in feet above the MLLW (mean lower low water) as a function of the time, x, in hours.

   a. How many periods are there each day?

$P = \frac{2\pi}{\omega}$ $\quad \omega = \frac{\pi}{3}$

$P = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6 \text{ hrs}$

If period is 6 hrs, there are four periods in a day.

   b. Write a function that gives the time in hours as a function of the water level. How many other times per day will have the same water levels as those given by the function? I need to reverse the function

$y = 1.236 \sin\left(\frac{\pi}{3}x\right) + 1.798$

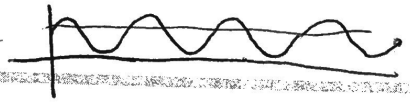
$y - 1.798 = 1.236 \sin\left(\frac{\pi}{3}x\right)$

$\frac{y-1.798}{1.236} = \sin\left(\frac{\pi}{3}x\right)$

$\frac{\pi}{3}x = \sin^{-1}\left(\frac{y-1.798}{1.236}\right)$

$x = \frac{3}{\pi}\sin^{-1}\left(\frac{y-1.798}{1.236}\right)$

Example:



most water levels happen 8 times per day.