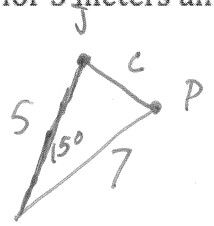
Law of Sines, $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Lesson 16: Trig Review Week

Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos(A)$

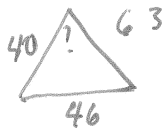
1. During a figure skating routine, Jackie and Peter skate apart with an angle of 15° between them. Jackie skates for 5 meters and Peter skates for 7 meters. How far apart are the skaters?



$c^2 = 5^2 + 7^2 - 2(5)(7)\cos(15)$

$c = 2.527$ m

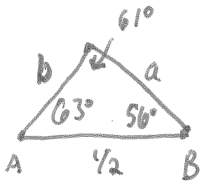
2. A bridge is supported by triangular braces. If the sides of each brace have lengths 63 feet, 46 feet, and 40 feet, find the measure of the angle opposite the 46 foot side.



$46^2 = 40^2 + 63^2 - 2(40)(63)\cos(A)$

$A = 46.755°$

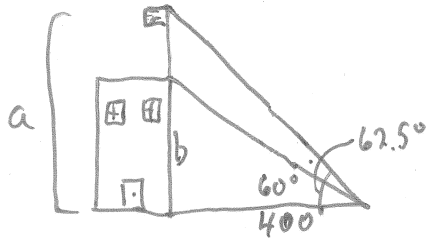
3. Two observers are standing on shore ½ mile apart at points A and B and measure the angle to a sailboat at a point C at the same time. Angle A is 63° and angle B is 56°. Find the distance from each observer to the sailboat.



$\frac{\frac{1}{2}}{\sin(61)} = \frac{a}{\sin(63)}$ $\quad a = 0.509$ mi

$\frac{\frac{1}{2}}{\sin(61)} = \frac{b}{\sin(56)}$ $\quad b = 0.474$ mi

4. A vertical flagpole is attached to the top edge of a building. A man stands 400 feet from the base of the building. From his viewpoint, the angle of elevation to the bottom of the flagpole is 60°; to the top is 62.5°. Determine the height of the flagpole.



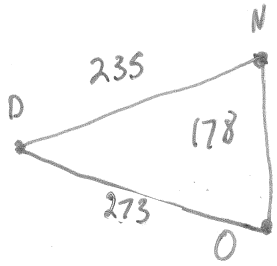
$\tan(62.5) = \frac{a}{400}$ $\quad 400\tan(62.5) = a$ $\quad a = 768.393$

$\tan(60) = \frac{b}{400}$ $\quad 400\tan(60) = b$ $\quad b = 692.820$

$a - b = 75.573$ ft

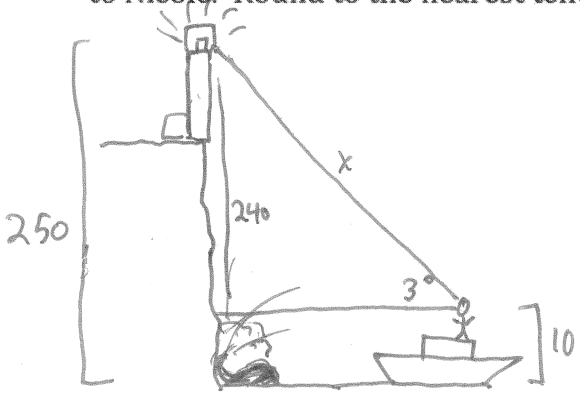
5. On a map, Orlando is 178 mm due south of Niagara Falls, Denver is 273 mm from Orlando, and Denver is 235 mm from Niagara Falls. Find the angle at Niagara Falls.

$273^2 = 235^2 + 178^2 - 2(273)(178)\cos(N)$



$c = 81.49^\circ$

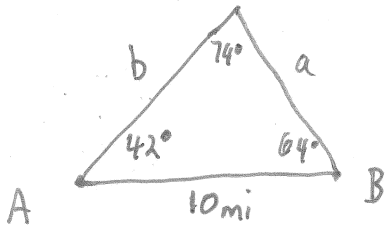
6. Nicole shines a light from a window of a lighthouse on a cliff 250 feet above the water level. Nick, 10 feet above the water level in a ship off shore, finds that the angle of elevation of the light is 3°. Find the length of the line of sight (light beam) from the ship to Nicole. Round to the nearest tenth.



$\sin(3^\circ) = \frac{240}{x}$

$x = \frac{240}{\sin(3^\circ)} = 4585.757\text{ ft}$

7. Fire towers A and B are located 10 miles apart. Rangers at fire tower A spot a fire at 42°, and rangers at fire tower B spot the same fire at 64°. How far from tower A is the fire to the nearest tenth of a mile?



$\frac{10}{\sin(74^\circ)} = \frac{b}{\sin(64)}$

$b = 9.350\text{ mi}$

Evaluate the given expression without the aid of a calculator.

1. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
2. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$
3. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$
4. $\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$
5. $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$
6. $\arctan(1) = \frac{\pi}{4}$
7. $\arcsin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
8. $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$
9. $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$
10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$
11. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$
12. $\tan^{-1}(-1) = -\frac{\pi}{4}$
13. $\sin^{-1}0 = 0$
14. $\cos^{-1}0 = \frac{\pi}{2}$
15. $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

Remember:

$\sin^{-1}(x)$ must choose from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(x)$ must choose from $[0, \pi]$

$\tan^{-1}(x)$ must choose from $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Solve each of the following equations for its principle value. NO CALCULATOR.

1) $1 = 3 + 4\cos\theta$

$-2 = 4\cos(\theta)$

$-\frac{1}{2} = \cos(\theta)$

$\cos^{-1}(-\frac{1}{2}) = \theta$

$\theta = \frac{2\pi}{3}$

2) $-3 - 4\tan\theta = 1$

$-4\tan(\theta) = 4$

$\tan(\theta) = -1$

$\tan^{-1}(-1) = \theta$

$\theta = -\frac{\pi}{4}$

3) $-4 + 6\cos\theta = -7$

$6\cos(\theta) = -3$

$\cos(\theta) = -\frac{1}{2}$

$\theta = \cos^{-1}(-\frac{1}{2})$

$\theta = \frac{2\pi}{3}$

4) $1 = -3 + 8\sin\theta$

$4 = 8\sin(\theta)$

$\frac{1}{2} = \sin(\theta)$

$\sin^{-1}(\frac{1}{2}) = \theta$

$\theta = \frac{\pi}{6}$

5) $1 + 3\sin\theta = 1$

$3\sin(\theta) = 0$

$\sin(\theta) = 0$

$\sin^{-1}(0) = \theta$

$\theta = 0$

6) $1 - 3\cos\theta = 1$

$-3\cos(\theta) = 0$

$\cos(\theta) = 0$

$\cos^{-1}(0) = \theta$

$\theta = \frac{\pi}{2}$

7) $-5 + \frac{2}{3} \cdot \cos\theta = -\frac{16}{3}$

$\frac{2}{3}\cos(\theta) = -\frac{16}{3} + 5$

$\frac{2}{3}\cos(\theta) = -\frac{16}{3} + \frac{15}{3}$

$\frac{2}{3}\cos(\theta) = -\frac{1}{3}$

$\cos(\theta) = -\frac{1}{3} \cdot \frac{3}{2}$

$\cos(\theta) = -\frac{1}{2}$ $\theta = \cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$

8) $-4 + \frac{1}{3} \cdot \sin\theta = -4$

$\frac{1}{3}\sin(\theta) = 0$

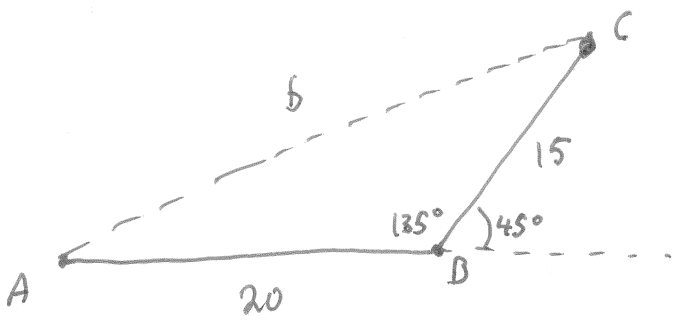
$\sin(\theta) = 0$

$\sin^{-1}(0) = \theta = 0$

1. Beatrice is standing 20 meters directly east of Ari, and Cece is standing 15 meters directly northeast of Beatrice.

   a. To one decimal place, what is the distance between Ari and Cece?

Directly NE means HALFWAY between N and E



$b^2 = 20^2 + 15^2 - 2 \cdot 20 \cdot 15 \cdot \cos(135°)$

$b = 32.4$ m.

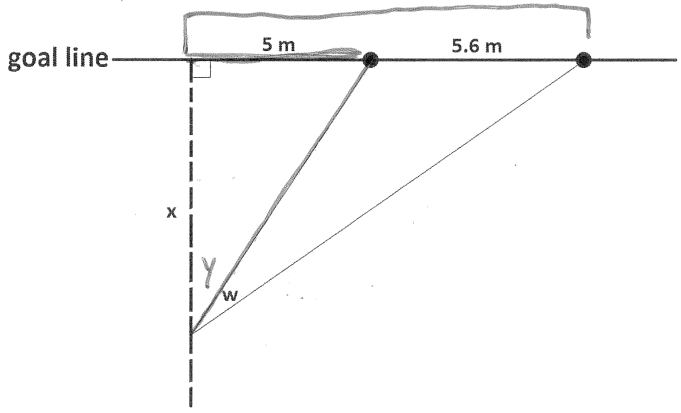
   b. To one decimal place, what is the measure of the smallest angle in the triangle formed by Ari, Beatrice, and Cece?

$$\frac{\sin(A)}{15} = \frac{\sin(135)}{32.4}$$

$$A = \sin^{-1}\left(\frac{15\sin(135)}{32.4}\right) \approx 19.1°$$

2. The diagram shows part of a rugby union football field. The goal line (marked) passes through two goal posts (marked as black circles) set 5.6 meters apart.

10.6



According to the rules of the game, an attempt at a conversion must be taken at a point on a line through the point of touchdown and perpendicular to the goal line. If a touchdown occurred 5 meters to one side of a goal post on the goal line, for example, the dashed line in the diagram indicates the line on which the conversion must be attempted.

Suppose the conversion is attempted at a distance of x meters from the goal line. Let w be the angle (measured in radians) indicated subtended by the goal posts.

a. Using inverse trigonometric functions, write an expression for w in terms of the distance x.

I'll label "y" on the diagram too ∴ use Two Right triangles

$\tan(y+w) = \frac{10.6}{x}$ $\quad \tan(y) = \frac{5}{x}$

$y+w = \tan^{-1}\left(\frac{10.6}{x}\right)$ $\quad y = \tan^{-1}\left(\frac{5}{x}\right)$

$y+w-y = \boxed{\tan^{-1}\left(\frac{10.6}{x}\right) - \tan^{-1}\left(\frac{5}{x}\right) = y}$