

Lesson 17: Trig Review Week Day 2

Find the first TWO solutions to the equation for $x > 0$. All answers should be in radians.

<p>1. $16 \sin(x) = 8$</p> <p>$\sin(x) = \frac{1}{2}$</p> <p>$x = \frac{\pi}{6}$</p> <p>$x = \frac{5\pi}{6}$</p>	<p>5. $20 \sin(3x) - 2 = 5$</p> <p>$20 \sin(3x) = 7$</p> <p>$\sin(3x) = \frac{7}{20}$</p> <p>$3x = 0.358$ $3x = 2.784$</p> <p>$x = 0.119$ $x = 0.928$</p>
<p>2. $8 \cos(x) = 4\sqrt{3}$</p> <p>$\cos(x) = \frac{\sqrt{3}}{2}$</p> <p>$x = \frac{\pi}{6}$</p> <p>$x = \frac{11\pi}{6}$</p>	<p>6. $218 \cos\left(\frac{1}{2}x\right) + 2 = 17$</p> <p>$218 \cos\left(\frac{1}{2}x\right) = 15$</p> <p>$\cos\left(\frac{1}{2}x\right) = \frac{15}{218}$</p> <p>$\frac{1}{2}x = 1.502$ $\frac{1}{2}x = 4.781$</p> <p>$x = 3.004$ $x = 9.562$</p>
<p>3. $3 \cos(x) - 2 = -3.1$</p> <p>$3 \cos(x) = -1.1$</p> <p>$\cos(x) = -\frac{1.1}{3}$</p> <p>$x = 1.946$</p> <p>$x = 4.337$</p>	<p>7. $3 \cos(4.3x) - 2 = -1$</p> <p>$3 \cos(4.3x) = 1$</p> <p>$\cos(4.3x) = \frac{1}{3}$</p> <p>$4.3x = 1.231$ $4.3x = 5.052$</p> <p>$x = 0.286$ $x = 1.175$</p>
<p>4. $2.1 \sin(x) + \sqrt{5} = 3$</p> <p>$2.1 \sin(x) = 3 - \sqrt{5}$</p> <p>$\sin(x) = \frac{3 - \sqrt{5}}{2.1}$</p> <p>$x = 0.372$</p> <p>$x = 2.769$</p>	<p>8. $1200 \sin(4x) + 130 = 100$</p> <p>$1200 \sin(4x) = -30$</p> <p>$\sin(4x) = \frac{-30}{1200}$</p> <p>$4x = -0.025$ $4x = 3.167$</p> <p>$4x = 2\pi - 0.025$ $x = 0.792$</p> <p>$x = -1.565$</p>

The population of turkeys in a state park is modeled by the function $T(t)$ where t is measured in years.

$$T(t) = 95 \cos\left(\frac{\pi}{8}t\right) + 150.$$

- a. Identify the amplitude, the period, and the midline of model and explain what those numbers mean in the context of the situation.

Amplitude: 95 : the population varies by 95 turkeys from its average

Period: $\frac{2\pi}{\frac{\pi}{8}} = 16$ yrs. It takes 16 years for the population of turkeys to go through its full decline cycle.

Midline: 150 : The average turkey population is 150

- b. When will the population of turkeys first reach 200.

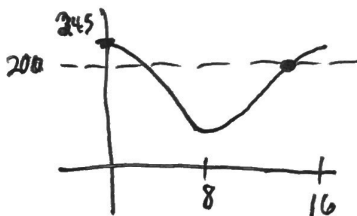
$$200 = 95 \cos\left(\frac{\pi}{8}t\right) + 150$$

$$\frac{50}{95} = \cos\left(\frac{\pi}{8}t\right)$$

$$\frac{\pi}{8}t = 1.017$$

$$t = 2.589 \text{ yrs}$$

- c. When will the population of turkeys reach 300 for the second time? At this time, is the population of turkeys increasing or decreasing?



Already found

$$\frac{\pi}{8}t = 1.017$$

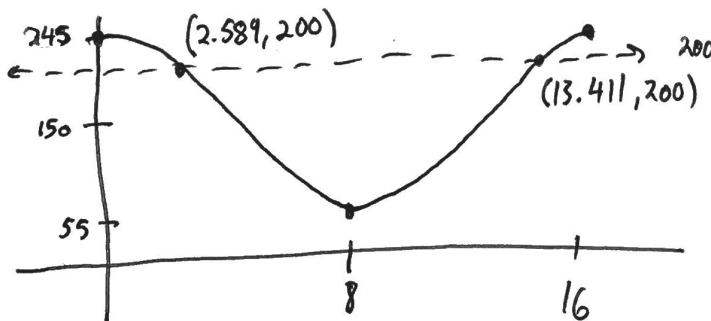
$$t = 2.589 \text{ yrs}$$

$$\frac{\pi}{8}t = 5.267$$

$$t = 13.411$$

Population is increasing

- d. Sketch the model and draw a horizontal line that represents the population at 300 turkeys.



The length of a spring as it bounces is modeled by $S(t)$ where t is measured in seconds and S is in feet.
 $S(t) = 2 \cos(2\pi t) + 2.9$.

- a. Identify the amplitude, the period, and the midline of model and explain what those numbers mean in the context of the situation.

Amplitude: 2 ft: the variation in the length of the ~~middle~~ spring from the midline.

Period: $\frac{2\pi}{2\pi} = 1$ second. This is how long it takes the spring to compress, extend, and the compress again.

Midline: 2.9 ft: the average length of the spring.

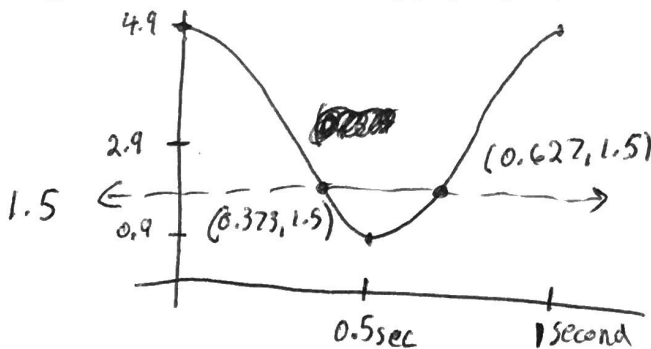
- b. When will the spring first reach a length of 1.5 feet?

$$\begin{aligned} 1.5 &= 2 \cos(2\pi t) + 2.9 \\ -1.4 &= 2 \cos(2\pi t) \\ -0.7 &= \cos(2\pi t) \\ 2\pi t &= \cos^{-1}(-0.7) = 2.346 \\ t &= \frac{2.346}{2\pi} \approx 0.373 \text{ sec} \end{aligned}$$

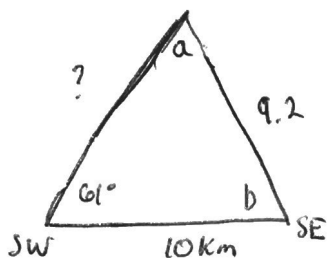
- c. When will the spring reach a length of 1.5 feet for the second time? At this time, is the ~~population~~ ^{length of the spring} ~~of turkeys~~ increasing or decreasing?

$$\begin{aligned} \text{If } 2\pi t &= \cos^{-1}(-0.7) = 2.346 \\ 2\pi - 2.346 &= 3.937 \\ 2\pi t &= 3.937 \\ t &= \frac{3.937}{2\pi} \approx 0.627 \text{ sec the length is increasing!} \end{aligned}$$

- d. Confirm these two times by graphing on your calculator.



1. Two ranger stations located 10 km apart on the southwest and southeast corners of a national park. They receive a distress call from a camper. Electronic equipment allows SW ranger to determine that the camper is at a location that makes an angle of 61° with the southern boundary. Another beacon allows the SE ranger see that the camper is 9.2 km to the northwest of his position.
 (a) Which station is closer to the camper? (b) What is the difference in the distances?



SE station is closer by:
 $9.2 - 7.702 = 1.498$ km

$$\frac{\sin(61)}{9.2} = \frac{\sin(a)}{10}$$

$$\frac{10 \sin(61)}{9.2} = \sin(a)$$

$$a = 71.929$$

that means b is $180 - 61 - 71.929 = 47.071$

$$\frac{?}{\sin(47.071)} = \frac{9.2}{\sin(61)}$$

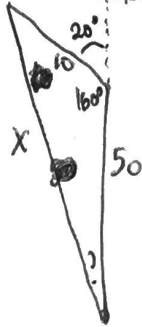
$$? = \frac{9.2 \sin(47.071)}{\sin(61)}$$

$$? = 7.702$$

2. Ships A and B leave port at the same time and sail on straight paths making an angle of 60° with each other. How far apart are the ships at the end of 1 hour if the speed of ship A is 25 km/h and that of ship B is 15 km/h?

3. A plane flies 500 miles on a straight path. The plane then turns left 12 degrees on a new heading and goes another 300 miles. How far is the plane from its original location?

4. A boat leaves a pier heading due north for 50 miles. The captain then turns 20° toward the west and goes another 10 miles. At this point the boat breaks down. What angle (from north) does the harbor need to send a tug boat to retrieve the boat and the captain?



$$x^2 = 10^2 + 50^2 - 2(10)(50)\cos(160)$$

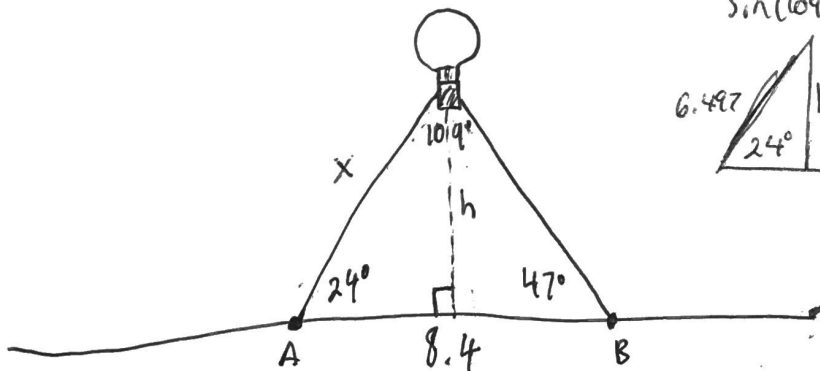
$$x = 59.495 \text{ mi}$$

$$\frac{59.495}{\sin(160)} = \frac{10}{\sin(?)}$$

$$\sin(?) = \frac{10 \sin(160)}{59.495}$$

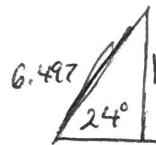
$$? = \sin^{-1}\left(\frac{10 \sin(160)}{59.495}\right) = \boxed{3.296^\circ}$$

5. The angles of elevation of a balloon from the two points A and B on level ground are 24° and 47° respectively. If points A and B are 8.4 miles apart and the balloon is between the points, in the same vertical plane, approximate, to the nearest tenth of a mile, the height of the balloon above the ground.



$$\frac{8.4}{\sin(109)} = \frac{x}{\sin(47)}$$

$$\frac{\sin(47)(8.4)}{\sin(109)} = x = 6.497$$



$$\sin(24) = \frac{h}{6.497}$$

$$6.497 \cdot \sin(24) = h$$

$$h = \boxed{2.643 \text{ mi}}$$