Integration

1. $\dfrac{dy}{dt} = 3t^2$ ✓ $y' = 3t^2$ ✓

$\int dy = \int 3t^2\,dt$

$\boxed{y = t^3 + C}$

2. $\dfrac{dr}{d\theta} = \pi$ ✓ $r' = \pi$ ✓

$\int dr = \int \pi\,d\theta$

$\boxed{r = \pi\theta + C}$

3. $\dfrac{dy}{dx} = x^{3/2}$ ✓ $y' = \dfrac{2}{5}\left(\dfrac{5}{2}\right)x^{5/2 - 2/2} = \boxed{x^{3/2}}$ ✓

$\int dy = \int x^{3/2}\,dx$

$\boxed{y = \dfrac{2x^{5/2}}{5} + C}$

4. $\dfrac{dy}{dx} = 3x^{-4}$ $\qquad y' = -(-3)x^{-4} = 3x^{-4}$ ✓

$\int dy = \int 3x^{-4}\,dx$

$\boxed{y = \dfrac{3x^{-3}}{-3} = -x^{-3} + C}$

5. $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \boxed{\dfrac{3x^{4/3}}{4} + C}$

6. $\int \dfrac{1}{x^2}\,dx = \int x^{-2}\,dx = \dfrac{x^{-1}}{-1} = \boxed{\dfrac{-1}{x} + C}$

7. $\int \dfrac{1}{x\sqrt{x}}\,dx = \int \dfrac{1}{x\,x^{1/2}}\,dx = \int \dfrac{1}{x^{3/2}}\,dx = \int x^{-3/2}\,dx = \dfrac{x^{-1/2}}{-1/2} = -2x^{-1/2} = \boxed{\dfrac{-2}{\sqrt{x}} + C}$

8. $\int x(x^2+3)dx = \int(x^3+3x)dx = \boxed{\dfrac{x^4}{4}+\dfrac{3x^2}{2}+C}$

9. $\int \dfrac{1}{2x^3}dx = \dfrac{1}{2}\int x^{-3}dx = \dfrac{1}{2}\dfrac{x^{-2}}{-2} = -\dfrac{1}{4}x^{-2} = \boxed{\dfrac{-1}{4x^2}+C}$

10. $\int \dfrac{1}{(2x)^3}dx = \int \dfrac{1}{8x^3}dx = \dfrac{1}{8}\int x^{-3}dx = \dfrac{1}{8}\dfrac{x^{-2}}{-2} = \dfrac{-1}{16}x^{-2} = \boxed{\dfrac{-1}{16x^2}+C}$

11. $\int(x^3+2)dx = \boxed{\dfrac{x^4}{4}+2x+C}$ ✓: $y' = \dfrac{4}{4}x^3+2 = x^3+2$ ✓

12. $\int(x^2-2x+3)dx = \dfrac{x^3}{3}-\dfrac{2x^2}{2}+3x+C$ ✓: $y'=3\left(\dfrac{1}{3}\right)x^2-2x+3$

$\boxed{=\dfrac{1}{3}x^3-x^2+3x+C}$ $y'=x^2-2x+3$

13. $\int(x^{3/2}+2x+1)dx = \dfrac{2x^{5/2}}{5}+\dfrac{2x^2}{2}+x+C$ ✓: $y'=\dfrac{2}{5}\left(\dfrac{5}{2}\right)x^{3/2}+2x+1$

$\boxed{F(x)=\dfrac{2}{5}x^{5/2}+x^2+x+C}$ $y'=x^{3/2}+2x+1$

14. $\int\left(\sqrt{x}+\dfrac{1}{2\sqrt{x}}\right)dx = \int\left(x^{1/2}+\dfrac{1}{2}x^{-1/2}\right)dx$ ✓: $y'=\dfrac{2}{3}\left(\dfrac{3}{2}\right)x^{1/2}+\dfrac{1}{2}x^{-1/2}$

$= \dfrac{2x^{3/2}}{3}+\dfrac{1}{2}x^{1/2}(2)+C$ $y'=x^{1/2}+\dfrac{1}{2\sqrt{x}}$ ✓

$\boxed{=\dfrac{2}{3}x^{3/2}+x^{1/2}+C}$

15. $\int\sqrt[3]{x^2}dx = \int x^{2/3}dx = \boxed{\dfrac{3x^{5/3}}{5}+C}$ ✓: $y'=\dfrac{3}{5}\left(\dfrac{5}{3}\right)x^{2/3}=\sqrt[3]{x^2}$ ✓

16. $\int \left(\sqrt[4]{x^3} + 1\right)dx = \int \left(x^{3/4} + 1\right)dx = \boxed{\dfrac{4x^{7/4}}{7} + x + C} \;\rightarrow\; y' = \dfrac{7}{4}\left(\dfrac{4}{7}\right)x^{3/4} + 1$

$y' = x^{3/4} + 1 \;\checkmark$

17. $\int \dfrac{1}{x^3}\,dx = \int x^{-3}\,dx = \dfrac{x^{-2}}{-2} + C = \boxed{\dfrac{-1}{2x^2} + C} \;\rightarrow\; \checkmark : y' = \dfrac{-2}{-2}x^{-3} = \dfrac{1}{x^3} \;\checkmark$

18. $\int \dfrac{1}{x^4}\,dx = \int x^{-4}\,dx = \dfrac{x^{-3}}{-3} + C = \boxed{\dfrac{-1}{3x^3} + C} \;\rightarrow\; y' = \dfrac{-3}{-3}x^{-4} = x^{-4} = \dfrac{1}{x^4} \;\checkmark$

19. $\int \dfrac{x^2+x+1}{\sqrt{x}}\,dx = \int \left(\dfrac{x^2}{x^{1/2}} + \dfrac{x^1}{x^{1/2}} + \dfrac{1}{x^{1/2}}\right)dx = \int \left(x^{3/2} + x^{1/2} + x^{-1/2}\right)dx =$

$y = \boxed{\dfrac{2x^{5/2}}{5} + \dfrac{2x^{3/2}}{3} + 2x^{1/2} + C} \;\rightarrow\; \checkmark : y' = \dfrac{2}{5}\left(\dfrac{5}{2}\right)x^{3/2} + \dfrac{2}{3}\left(\dfrac{3}{2}\right)x^{1/2} + 2\left(\dfrac{1}{2}\right)x^{-1/2}$

$y' = x^{3/2} + x^{1/2} + x^{-1/2}$

20. $\int \dfrac{x^2+1}{x^2}\,dx = \int \left(\dfrac{x^2}{x^2} + \dfrac{1}{x^2}\right)dx = \int \left(1 + x^{-2}\right)dx = x + \dfrac{x^{-1}}{-1} + C = \boxed{x - \dfrac{1}{x} + C}$

$y' = 1 + x^{-2} = 1 + \dfrac{1}{x^2} \;\checkmark$

21. $\int (x+1)(3x-2)\,dx = \int \left(3x^2 + x - 2\right)dx = \dfrac{3x^3}{3} + \dfrac{x^2}{2} - 2x + C = \boxed{x^3 + \dfrac{1}{2}x^2 - 2x + C}$

$y' = 3x^2 + \dfrac{1}{2}(2)x - 2 \;\checkmark$

22. $\int (2t^2-1)^2\,dt = \int (2t^2-1)(2t^2-1)\,dt = \int \left(4t^4 - 4t^2 + 1\right)dt =$

$\boxed{\dfrac{4t^5}{5} - \dfrac{4t^3}{3} + t + C}$ \quad check: $y' = 5\left(\dfrac{4}{5}\right)t^4 - 3\left(\dfrac{4}{3}\right)t^2 + 1$

$y' = 4t^4 - 4t^2 + 1 \;\checkmark$

23. $\int y^2 \sqrt{y}\, dy = \int y^2(y^{1/2})dy = \int y^{5/2}dy = \boxed{\dfrac{2y^{7/2}}{7}+C}$ $F'=\dfrac{2}{7}\left(\dfrac{7}{2}\right)y^{5/2}$ ✓

24. $\int (1+3t)t^2\, dt = \int(t^2+3t^3)dt = \boxed{\dfrac{t^3}{3}+\dfrac{3t^4}{4}+C}$ $F'=\dfrac{3}{3}t^2+4\left(\dfrac{3}{4}\right)t^3$

$F'=t^2+3t^3=t^2(1+3t)$

25. $\int dx = x+c$ ✓: $y'=1$

26. $\int 3dt = 3x+C$ ✓: $y'=3$ ✓

27. $\int (2\sin x+3\cos x)dx = \boxed{-2\cos x + 3\sin x +C}$

✓: $y'= +2\sin x + 3\cos x$ ✓

28. $\int (t^2-\sin t)dt = \boxed{\dfrac{t^3}{3}+\cos t +C}$ $y'=\dfrac{3t^2}{3}-\sin t$ ✓

29. $\int (1-\csc t\cot t)dt = \boxed{t+\csc t +c}$ $y'=1-\csc t\cot t$ ✓

30. $\int (\theta^2+\sec^2\theta)d\theta = \boxed{\dfrac{\theta^3}{3}+\tan\theta +C}$ $y'=\dfrac{3\theta^2}{3}+\sec^2\theta$ ✓

31. $\int (\sec^2\theta - \sin\theta)d\theta = \boxed{\tan\theta + \cos\theta +C}$ $y'=\sec^2\theta -\sin\theta$ ✓

32. $\int \sec y(\tan y - \sec y)dy = \int(\sec y\tan y - \sec^2 y)dy = \boxed{\sec y - \tan y +C}$

✓: $F'=\sec y\tan y -\sec^2 y$ ✓

33. $\int (\tan^2 y +1)dy = \int \sec^2 y\, dy = \boxed{\tan y +C}$

$\dfrac{\sin^2 + \cos^2 =1}{\cos^2}$ $\tan^2 +1 = \sec^2$

34. $\int \frac{\sin x}{1-\sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx = \int \tan x \cdot \sec x \cdot dx$
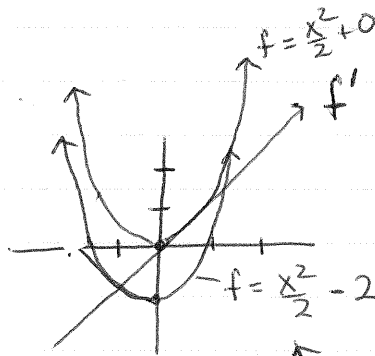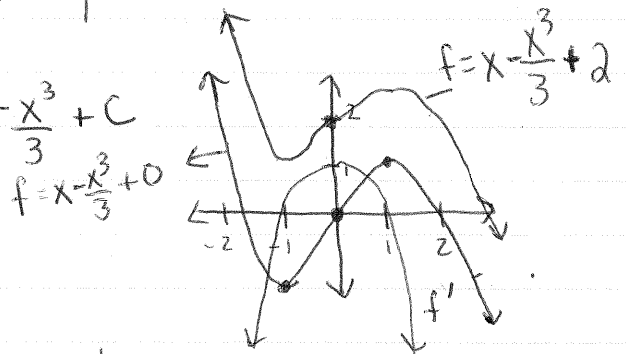
$\sin^2 + \cos^2 = 1$

$\cos^2 = 1 - \sin^2$

$\longrightarrow \boxed{= \sec x + C}$

37. $f' = 2 \quad f = \int f' = \int 2\,dx$

so $f = 2x + C$

$f = 2x + 0$
$f = 2x - 2$
f'

38. $f' = x \quad f = \int f' = \int x\,dx = \frac{x^2}{2} + C$

$f = \frac{x^2}{2} + 0$
f'
$f = \frac{x^2}{2} - 2$

39. $f' = 1 - x^2 \quad f = \int f' = \int (1-x^2)\,dx = x - \frac{x^3}{3} + C$

$f = x - \frac{x^3}{3} + 0$

$f = x - \frac{x^3}{3} + 2$
f'

40. $f' = \frac{1}{x^2} \quad f = \int f' = \int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + C$

$f = -\frac{1}{x} + C$

f'
$f = -\frac{1}{x} + 3$

$f = -\frac{1}{x} - 3$