

4. Consider the case of \overline{CA} and \overline{CB} of lengths 5 and 12, respectively, with $m\angle C > 180^\circ$.

- a. Is the law of cosines consistent in being able to calculate the length of \overline{AB} even using an angle this large? Try it for $m\angle C = 200^\circ$, and compare your results to the triangle with $m\angle C = 160^\circ$. Explain your findings.

Yes, the law of cosines is still able to calculate the length of \overline{AB} :

$$c^2 = 5^2 + 12^2 - 2 \cdot 5 \cdot 12 \cos(200^\circ) \\ \approx 281.76$$

Thus, $c \approx 16.786$, which represents the length of line segment AB . We get the same result for $\cos(160^\circ)$, which makes sense since it is true that $\cos(\theta) = \cos(360^\circ - \theta)$.

- b. Consider what you have learned in Problems 1–4. If you were designing a computer program to be able to measure sides and angles of triangles created from different line segments and angles, would it make sense to use the law of cosines or several different techniques depending on the shape? Would a computer program created from the law of cosines have any errors based on different inputs for the line segments and angle between them?

The benefits of using the law of cosines would be that there would be no need for logic involving different cases to be programmed, and there would be no exceptions to the formula. The law of cosines would work even when the shape formed was not a triangle or if the shape was formed using an angle greater than 180° . The triangle inequality theorem would need to be used to verify whether the side lengths could represent those of a triangle. There would be no errors for two line segments and the angle between them.

5. Consider triangles with the following measurements. If two sides are given, use the law of cosines to find the measure of the third side. If three sides are given, use the law of cosines to find the measure of the angle between a and b .

- a. $a = 4, b = 6, m\angle C = 35^\circ$

$$c \approx 3.56$$

- b. $a = 2, b = 3, m\angle C = 110^\circ$

$$c \approx 4.14$$

- c. $a = 5, b = 5, m\angle C = 36^\circ$

$$c \approx 3.09$$

- d. $a = 7.5, b = 10, m\angle C = 90^\circ$

$$c \approx 12.5$$

- e. $a = 4.4, b = 6.2, m\angle C = 9^\circ$

$$c \approx 1.98$$

- f. $a = 12, b = 5, m\angle C = 45^\circ$

$$c \approx 9.17$$

- g. $a = 3, b = 6, m\angle C = 60^\circ$

$$c \approx 5.2$$

h. $a = 4, b = 5, c = 6$

$m\angle C \approx 82.82^\circ$

i. $a = 1, b = 1, c = 1$

$m\angle C = 60^\circ$

j. $a = 7, b = 8, c = 3$

$m\angle C \approx 21.79^\circ$

k. $a = 6, b = 5.5, c = 6.5$

$m\angle C \approx 68.68^\circ$

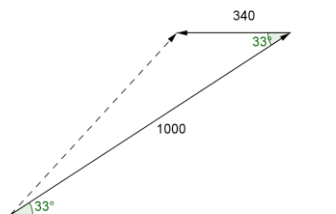
l. $a = 8, b = 5, c = 12$

$m\angle C \approx 133.43^\circ$

m. $a = 4.6, b = 9, c = 11.9$

$m\angle C \approx 118.45^\circ$

6. A trebuchet launches a boulder at an angle of elevation of 33° at a force of 1,000 N. A strong gale wind is blowing against the boulder parallel to the ground at a force of 340 N. The figure is shown below.



- a. What is the force in the direction of the boulder's path?

Since the wind is blowing in the opposite direction, finding the sum of the two vectors is similar to finding the difference between two vectors if the wind is in the same direction. Thus, we are finding the third side of a triangle (the diagonal of the parallelogram).

$$c^2 = 1000^2 + 340^2 - 2 \cdot 1000 \cdot 340 \cos(33^\circ)$$

$$\approx 738.45$$

The boulder is traveling with an initial force of 738.45 N.

- b. What is the angle of elevation of the boulder after the wind has influenced its path?

The angle between the original trajectory and the new trajectory is 14.52° , so the new angle of elevation is 47.52° .