7.10 Exponential Models Name:____________ Date:__________

Exponential Models

Example: A population p can be modeled by the function $P = Ce^{kt}$ where C and k are both constants. If the population doubles every 5 years, what is are the two constants?

$P = Ce^{kt}$

(t, p)

$(0, 1)$

$(5, 2)$

$1 = Ce^{k(0)}$

$1 = Ce^{0}$

$\boxed{C = 1}$

$2 = e^{k(5)}$

$\ln(2) = k(5)$

$\boxed{\frac{\ln(2)}{5} = k}$

$P = e^{\frac{\ln(2)}{5}t}$

Example: At the beginning of the summer, the population of a hive of bald-faced hornets is growing at a rate proportional to the population. The population model follows the equation: $P = Ce^{kt}$. From a population of 10 on the morning of May 1, the number of hornets grows to 50 in 30 days. If the growth continues to follow the same model, how many days after May 1 will the population reach 100?

(t, P)

$(0, 10)$

$(30, 50)$

$P = Ce^{kt}$

$10 = Ce^{0}$

$C = 10$

$P = 10e^{kt}$

$50 = 10e^{30k}$

$5 = e^{30k}$

$\ln(5) = 30k$

$\frac{\ln(5)}{30} = k$

$P = 10e^{\frac{\ln(5)}{30}t}$

$100 = 10e^{\frac{\ln(5)}{30}t}$

$10 = e^{\ln(5)/30\, t}$

$\ln(10) = \frac{\ln(5)}{30}t$

$\frac{30\ln(10)}{\ln(5)} = t$

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Practice: The decay of a specific isotope can be modeled by the function $A = Ce^{-kt}$ where C and k are both constants. If the amount of the isotope halves every 30 years, find a specific model for A, the amount of the isotope.

$A = Ce^{-kt}$

$2 = Ce^{0}$

$C = 2$

$A = 2e^{-k(30)}$

$1 = 2e^{-k(30)}$

$\frac{1}{2} = e^{-k(30)}$

(t, A)

$(0, 2)$

$(30, 1)$

$\ln(\frac{1}{2}) = -k(30)$

$-\frac{1}{30}\ln(\frac{1}{2}) = k$

$\ln(\sqrt[30]{2}) = k$

$A = 2e^{-\ln(\sqrt[30]{2})t}$

Practice: A population of bacteria starts with 50 bacteria in a dish. The population model follows the equation: $P = Ce^{kt}$. Four hours later, there are 200 bacteria in the dish. How many bacteria are in the dish 10 hours later?

$P = Ce^{kt}$

$50 = Ce^{0}$

$C = 50$

$P = 50e^{kt}$

$200 = 50e^{4k}$

$4 = e^{4k}$

$\ln(4) = 4k$

$\frac{\ln(4)}{4} = k$

$\ln(\sqrt[4]{4}) = k$

(t, P)

$(0, 50)$

$(4, 200)$

$t = 10,\ P = ?$

$P = 50e^{\ln(\sqrt[4]{4})t}$

$P = 50e^{\ln(\sqrt[4]{4})10}$