

Advanced Problems

3.  $\int_1^2 \frac{1}{x^3} dx = \int_1^2 x^{-2} dx = -x^{-1} \Big|_1^2 = -\left[\frac{1}{2} - 1\right] = -\frac{1}{2} + 1 = \frac{1}{2}$

- (A)  $-\frac{1}{2}$       (B)  $\frac{7}{24}$       (C)  $\frac{1}{2}$       (D) 1      (E)  $2 \ln 2$

$\left(\frac{v^2-1}{x}\right) = \frac{v^2}{x} - \frac{1}{x} = x^{-\frac{1}{2}} - \frac{1}{x}$

7.  $\int_1^e \left(\frac{x^2-1}{x}\right) dx = \int_1^e \left(x - \frac{1}{x}\right) dx = \frac{1}{2}x^2 - \ln|x| \Big|_1^e = \left[\frac{1}{2}(e)^2 - \ln(e)\right] - \left[\frac{1}{2}(1)^2 - \ln(1)\right]$

- (A)  $e - \frac{1}{e}$       (B)  $e^2 - e$       (C)  $\frac{e^2}{2} - e + \frac{1}{2}$       (D)  $e^2 - 2$       (E)  $\frac{e^2}{2} - \frac{3}{2}$

$\frac{1}{2}e^2 - \frac{1}{2} - \frac{1}{2}$   
 $\frac{e^2}{2} - \frac{3}{2}$

20. What are all values of  $k$  for which  $\int_{-3}^k x^2 dx = 0$ ?  $\int_{-3}^k x^2 dx = \frac{1}{3}x^3 \Big|_{-3}^k = \frac{1}{3}k^3 - \frac{1}{3}(-3)^3$

- (A) -3      (B) 0      (C) 3      (D) -3 and 3      (E) -3, 0, and 3

$\frac{1}{3}k^3 + 9 = 0$   
 $\frac{1}{3}k^3 = -9$   
 $k^3 = -27$

82. If  $f$  is a continuous function and if  $F'(x) = f(x)$  for all real numbers  $x$ , then  $\int_1^3 f(2x) dx =$

- (A)  $2F(3) - 2F(1)$   
(B)  $\frac{1}{2}F(3) - \frac{1}{2}F(1)$   
(C)  $2F(6) - 2F(2)$   
(D)  $F(6) - F(2)$   
(E)  $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

$\frac{1}{2} \int_2^6 f(u) du = \frac{1}{2} F(u) \Big|_2^6 = \frac{1}{2} F(6) - \frac{1}{2} F(2)$