9.2 Practice Solutions

Chain rule of (8-t) gives the negative sign

#1

The first thing we'll need here are the following two derivatives.

$$\frac{dx}{dt} = 12t^{\frac{1}{2}} \qquad \frac{dy}{dt} = -\frac{3}{2}(8-t)^{\frac{1}{2}}$$

Hide Step 2 ▾

We'll need the ds for this problem.

$$ds = \sqrt{\left[12t^{\frac{1}{2}}\right]^2 + \left[-\frac{3}{2}(8-t)^{\frac{1}{2}}\right]^2}\,dt = \sqrt{144t + \frac{9}{4}(8-t)}\,dt = \sqrt{\frac{567}{4}t + 18}\,dt$$

Hide Step 3 ▾

The integral for the arc length is then,

$$L = \int ds = \int_0^4 \sqrt{\frac{567}{4}t + 18}\,dt$$

#2

The first thing we'll need here are the following two derivatives.

$$\frac{dx}{dt} = 3 \qquad \frac{dy}{dt} = -2t$$

Hide Step 2 ▾

We'll need the ds for this problem.

$$ds = \sqrt{[3]^2 + [-2t]^2}\,dt = \sqrt{9 + 4t^2}\,dt$$

Hide Step 3 ▾

The integral for the arc length is then,

$$L = \int ds = \int_{-2}^{0} \sqrt{9 + 4t^2}\,dt$$

Product rule, but sin(2t) also gives us a chain rule for the second part of the product rule

#3

The first thing we'll need here are the following two derivatives.

$$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = \mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)$$

Hide Step 2 ▾

We'll need the ds for this problem.

$$ds = \sqrt{[2t]^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)\right]^2}\, dt = \sqrt{4t^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)\right]^2}\, dt$$

Hide Step 3 ▾

The integral for the arc length is then,

$$L = \int ds = \int_0^3 \sqrt{4t^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)\right]^2}\, dt$$

This is a double chain of $\cos^3(2t)$. The third power is most outside, then the cos(_), then the 2t. The -6 comes from the 3 of the power, the negative of the cosine derivative, and the 2 of the 2t derivative.

Chain rule gives the negative sign

#4

The first thing we'll need here are the following two derivatives.

$$\frac{dx}{dt} = -6\cos^2(2t)\sin(2t) \qquad \frac{dy}{dt} = -2t\cos\left(1 - t^2\right)$$

Hide Step 2 ▾

We'll need the ds for this problem.

$$\begin{aligned} ds &= \sqrt{\left[-6\cos^2(2t)\sin(2t)\right]^2 + \left[-2t\cos\left(1-t^2\right)\right]^2}\, dt \\ &= \sqrt{36\cos^4(2t)\sin^2(2t) + 4t^2\cos^2\left(1-t^2\right)}\, dt \end{aligned}$$

Hide Step 3 ▾

The integral for the arc length is then,

$$L = \int ds = \int_{-\frac{3}{2}}^{0} \sqrt{36\cos^4(2t)\sin^2(2t) + 4t^2\cos^2\left(1-t^2\right)}\, dt$$