9.2 Practice Solutions

#1

The first thing we'll need here are the following two derivatives.

$$rac{dx}{dt}=12t^{rac{1}{2}} \qquad rac{dy}{dt}=-rac{3}{2}(8-t)^{rac{1}{2}}$$

Hide Step 2 ▼

We'll need the ds for this problem.

$$ds = \sqrt{\left[12t^{rac{1}{2}}
ight]^2 + \left[-rac{3}{2}(8-t)^{rac{1}{2}}
ight]^2} \, dt = \sqrt{144t + rac{9}{4}(8-t)} \, dt = \sqrt{rac{567}{4}t + 18} \, dt$$

Hide Step 3 ▼

The integral for the arc length is then,

$$L = \int\!\!ds = \int_0^4 \sqrt{rac{567}{4}t + 18}\,dt$$

#2

The first thing we'll need here are the following two derivatives.

$$rac{dx}{dt}=3 \qquad rac{dy}{dt}=-2t$$

Chain rule of (8-t) gives the negative

Hide Step 2 ▼

We'll need the ds for this problem.

$$ds=\sqrt{\left[3
ight]^{2}+\left[-2t
ight]^{2}}\,dt=\sqrt{9+4t^{2}}\,dt$$

Hide Step 3 ▼

The integral for the arc length is then,

$$L=\int\!\!ds=\int_{-2}^0\sqrt{9+4t^2}\,dt$$

The first thing we'll need here are the following two derivatives.

Product rule, but sin(2t), also gives us a chain rule for the second part of the product rule cos(2t)

$$rac{dx}{dt} = 2t \qquad rac{dy}{dt} = \mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)$$

Hide Step 2 ▼

We'll need the ds for this problem.

$$ds = \sqrt{\left[2t
ight]^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)
ight]^2} \, dt = \sqrt{4t^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)
ight]^2} \, dt$$

Hide Step 3 ▼

The integral for the arc length is then,

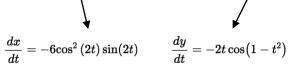
$$L = \int\!\!ds = \int_0^3 \sqrt{4t^2 + \left[\mathbf{e}^t \sin(2t) + 2\mathbf{e}^t \cos(2t)
ight]^2} \,dt$$

This is a double chain of $\cos^3(2t)$. The third power is most outside, then the $\cos(_)$, then the 2t. The -6 comes from the 3 of the power, the negative of the cosine derivative, and the 2 of the 2t derivative.

Chain rule gives the negative sign

#4

The first thing we'll need here are the following two derivatives.



Hide Step 2 ▼

We'll need the ds for this problem.

$$egin{aligned} ds &= \sqrt{\left[-6 ext{cos}^2\left(2t
ight) ext{sin}(2t)
ight]^2 + \left[-2t ext{cos}\left(1-t^2
ight)
ight]^2} \, dt \ &= \sqrt{36 ext{cos}^4\left(2t
ight) ext{sin}^2\left(2t
ight) + 4t^2 ext{cos}^2\left(1-t^2
ight)} \, dt \end{aligned}$$

Hide Step 3 ▼

The integral for the arc length is then,

$$L=\int\!\!ds=\int_{-rac{3}{2}}^{0}\sqrt{36\mathrm{cos}^{4}\left(2t
ight)\sin^{2}\left(2t
ight)+4t^{2}\mathrm{cos}^{2}\left(1-t^{2}
ight)}\,dt$$