Use the Ratio Test or the Alternating Series Test (AST) to determine if the series converges. If a test is inconclusive, use a different test to check convergence.

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)!} \cdot \underbrace{\frac{n!}{n^3}}_{\text{reciprocal of } \frac{n^3}{n!}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)\cdot n^3} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^3}{n^4+n^3} \right| = \lim_{n\to\infty} \left| \frac{1}{n} \right| = 0 < 1$$

↳ denom "wins"

$\sum_{n=1}^{\infty} \frac{n^3}{n!}$ converges by the ratio test

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{2}{n^2}$

$$\lim_{n\to\infty} \left| \frac{2}{(n+1)^2} \cdot \frac{n^2}{2} \right| = \lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \right| = \lim_{n\to\infty} \left| \frac{n^2}{n^2} \right| = 1$$

inconclusive!

$\frac{2}{n^2}$ looks like $\frac{1}{n^2}$; $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges p-series w/ $p > 1$

use Limit Comparison test

$$\lim_{n\to\infty} \frac{\frac{2}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{2}{n^2} \cdot \frac{n^2}{1} = \lim_{n\to\infty} 2 = 2 \qquad 0 < 2 < \infty$$

so $\sum_{n=1}^{\infty} \frac{2}{n^2}$ converges by the Limit Comparison Test

(k) AST: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \left[\frac{1}{n}\right]$

1. $\frac{1}{n} > 0$
2. $\frac{1}{n} > \frac{1}{n+1}$ The series eventually decreases
3. $\lim_{n \to \infty} \frac{1}{n} = 0$

The series converges by the Alternating Series Test

(l) AST: $\sum_{n=1}^{\infty} \frac{(-1)^{n}(n+3)}{2n}$

$= \sum_{n=1}^{\infty} (-1)^{n} \left[\frac{n+3}{2n}\right]$

This is $\frac{(n+1)+3}{2(n+1)}$

1. $\frac{n+3}{2n} > 0$
2. $\frac{(n+3)}{2n} > \frac{n+4}{2n+2}$

The factor of 2 in the denom makes the denom grow faster, so the terms decrease eventually.

3. $\lim_{n \to \infty} \frac{n+3}{2n} = \frac{1}{2}$

Alternating Series Test inconclusive

Try n^{th} term test: $\lim_{n \to \infty} \frac{(-1)^{n}(n+3)}{2n}$ DNE

Series diverges by the nth term test