

Homework

1. Calculate the average rate of change of $f(x) = x^2 - 1$ over the interval $[1, 4]$.

- (a) 8 (b) 6 (c) 5 (d) 12 (e) 4

$$\frac{f(4) - f(1)}{4 - 1} = \frac{15 - 0}{3} = 5$$

2. Given $f(x) = x^3 + 4$, determine $f'(5)$

- a. 379 b. 129 c. 0 d. 79

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e. 75

$$f'(x) = 3x^2$$

$$f'(5) = 3(25) = 75$$

1. $f(x) = \begin{cases} 3x^2 + 4x + 4, & x < 1 \\ 2x^3 + bx + c, & x \geq 1 \end{cases}$

If $f(x)$ is continuous and differentiable at $x=1$, then what are the values of a and b ?

Continuity:

$$3x^2 + 4x + 4 = 2x^3 + bx + c \text{ at } x=1$$

$$3 + 4 + 4 = 2 + b + c$$

$$11 = 2 + b + c$$

$$9 = b + c$$

c = 5

Differentiability:

$$6x + 4 = 6x^2 + b \text{ at } x=1$$

$$6 + 4 = 6 + b$$

4 = b

3. If $f(x) = 8\sqrt[4]{x}$, $f'(16) =$

$$f(x) = 8 \cdot x^{1/4}$$

$$f'(x) = 2 \cdot x^{-3/4}$$

$$f'(x) = \frac{2}{4\sqrt[4]{x^3}}$$

$$f'(16) = \frac{2}{4\sqrt[4]{16^3}} = \frac{2}{2^3} = \frac{1}{4}$$

4. If $f(x) = x^2 + \sqrt{x} + \frac{1}{x}$, then the slope of the curve $f(x)$ at $x=1$ is

$$f(x) = x^2 + x^{1/2} + x^{-1}$$

$$f'(x) = 2x + \frac{1}{2}x^{-1/2} - x^{-2}$$

$$f'(1) = 2 + \frac{1}{2} - 1 = \frac{3}{2}$$

5. Write the equation to the tangent and normal of $f(x) = x^3 - 2x - 1$ at $x=2$.

Point: $f(2) = 8 - 4 - 1 = 3 \rightarrow (2, 3)$ T: $y - 3 = 10(x - 2)$

Slope: $f'(x) = 3x^2 - 2$
 $f'(2) = 10$

N: $y - 3 = -\frac{1}{10}(x - 2)$

6. Write the equation to the tangent and normal of $f(x) = x^2 - 2\sqrt{x} - 1$ at $x=1$.

Point: $f(1) = 1^2 - 2\sqrt{1} - 1 = 1 - 2 - 1 = -2 \rightarrow (1, -2)$

Slope: $f'(x) = 2x - x^{-1/2}$

$f(x) = x^2 - 2x^{1/2} - 1$ $f'(1) = 2(1) - (1)^{-1/2} = 2 - 1 = 1$

T: $y + 2 = 1(x - 1)$

N: $y + 2 = -1(x - 1)$