3.1 HW

P. 160 #1-7 odd, 13-31 odd

#1. $y = \sin(3x+1)$

$\frac{dy}{dx} = \cos(3x+1) \cdot 3 = 3\cos(3x+1)$

#3. $y = \cos(\sqrt{3}x)$

$\frac{dy}{dx} = -\sin(\sqrt{3}x) = -\sqrt{3}\sin(\sqrt{3}x)$

#5. $y = \left(\frac{\sin(x)}{1+\cos(x)}\right)^2$

Derivative of the inside needs quotient rule!

$\frac{dy}{dx} = 2\left(\frac{\sin(x)}{1+\cos(x)}\right) \cdot \left[\frac{(1+\cos(x))\cos(x) - \sin(x)(-\sin(x))}{(1+\cos(x))^2}\right]$

$\frac{dy}{dx} = 2\left(\frac{\sin(x)}{1+\cos(x)}\right) \cdot \left[\frac{\cos(x) + \cos^2(x) + \sin^2(x)}{(1+\cos(x))^2}\right]$

Remember: $\cos^2(x) + \sin^2(x) = 1$

$\frac{dy}{dx} = 2\left(\frac{\sin(x)}{1+\cos(x)}\right) \cdot \left[\frac{\cos(x)+1}{(1+\cos(x))^2}\right] = \frac{2\sin(x)}{(1+\cos(x))^2}$

#7. $y = \cos(\sin(x))$

$\frac{dy}{dx} = -\sin(\sin(x)) \cdot \cos(x)$

#13 $y = (x+\sqrt{x})^{-2}$

$\frac{dy}{dx} = -2(x+\sqrt{x})^{-3} \cdot (1 + \frac{1}{2}x^{-1/2})$

#15 $y = \sin^{-5}(x) - \cos^3(x)$

$\frac{dy}{dx} = -5\sin^{-6}(x) \cdot \cos(x) - 3\cos^2(x) \cdot (-\sin(x))$

$\frac{dy}{dx} = -5\sin^{-6}(x) \cdot \cos(x) + 3\cos^2(x) \cdot \sin(x)$

#17 $y = \sin^3(x) \cdot \tan(4x)$ product rule BUT each factor has a chain!

$\frac{dy}{dx} = \sin^3(x) \cdot \sec^2(4x) \cdot 4 + 3\sin^2(x) \cdot \cos(x) \cdot \tan(4x)$

$\frac{dy}{dx} = 4\sin^3(x) \cdot \sec^2(4x) + 3\sin^2(x) \cdot \cos(x) \cdot \tan(4x)$

#19 $y = \frac{3}{\sqrt{2x+1}} = 3(2x+1)^{-1/2}$

$\frac{dy}{dx} = -\frac{3}{2}(2x+1)^{-3/2} \cdot 2 = \frac{-3}{\sqrt{(2x+1)^3}}$

#21. $y = \sin^2(3x-2)$ Advanced problem! Three chains!

$\frac{dy}{dx} = 2\sin(3x-2)\cdot\cos(3x-2)\cdot 3$ outside: Δ^2

$\frac{dy}{dx} = 6\sin(3x-2)\cdot\cos(3x-2)$ inside1: $\sin(\Delta)$

inside 2: $3x-2$

#23. $y = (1+\cos^2(7x))^3$ Advanced problem! Four Chains

$\frac{dy}{dx} = 3(1+\cos^2(7x))^2\cdot[2\cos(7x)]\cdot[-\sin(7x)]\cdot 7$ outside: Δ^3

$\frac{dy}{dx} = -21(1+\cos^2(7x))^2\cdot[2\cos(7x)]\cdot[\sin(7x)]$ inside1: $1+\Delta^2$

$\frac{dy}{dx} = -42(1+\cos^2(7x))^2\cdot[\cos(7x)]\cdot[\sin(7x)]$ inside 2: $\cos(\Delta)$

inside 3: $7x$

#25. $r = \tan(2-\theta)$

$\frac{dr}{d\theta} = \sec^2(2-\theta)\cdot[-1] = -\sec^2(2-\theta)$

#27. $r = \sqrt{\theta\sin(\theta)} = (\theta\sin(\theta))^{1/2}$ chain w/ a product inside.

$\frac{dr}{d\theta} = \frac{1}{2}(\theta\sin(\theta))^{-1/2}\cdot[\theta\cos(\theta) + 1\cdot\sin(\theta)]$

$\frac{dr}{d\theta} = \frac{1}{2}(\theta\sin(\theta))^{-1/2}\cdot[\theta\cos(\theta) + \sin(\theta)] = \frac{\theta\cos(\theta)+\sin(\theta)}{2\sqrt{\theta\sin(\theta)}}$

#29. $y = \tan(x)$

$y' = \sec^2(x)$

$y'' = 2\sec(x)\cdot[\sec(x)\tan(x)] = 2\sec^2(x)\tan(x)$

#31 $y = \cot(3x-1)$

$y' = -\csc^2(3x-1)\cdot 3 = -3\csc^2(3x-1)$ $\longrightarrow$ for the second derivative, there are three chains!

$y'' = -6\csc(3x-1)\cdot[-\csc(3x-1)\cdot\cot(3x-1)]\cdot 3$

$y'' = 18\csc^2(3x-1)\cot(3x-1)$

Outside: $-3\Delta^2$

inside1: $\csc(\Delta)$

inside 2: $3x-1$