HW 3.3: Pg. 170 #1-19 odd, 23-29 odd, 55

(1) $x^2y + xy^2 = 6 \rightarrow (x^2\frac{d}{dx}y + y\frac{d}{dx}x^2) + (x\frac{d}{dy}y^2 + y^2\frac{d}{dx}x) = 0$

$x^2 \cdot \frac{dy}{dx} + 2xy + 2xy \cdot \frac{dy}{dx} + y^2 = 0$

$x^2 \cdot \frac{dy}{dx} + 2xy \cdot \frac{dy}{dx} = -y^2 - 2xy$

$\frac{dy}{dx}(x^2 + 2xy) = -y^2 - 2xy$

$\frac{dy}{dx} = \boxed{\frac{-y^2 - 2xy}{x^2 + 2xy}}$

(3) $y^2 = \frac{x-1}{x+1} \rightarrow \frac{dy}{dx}y^2 = \frac{(x+1)\frac{dy}{dx}(x-1) - (x-1)\frac{dy}{dx}(x+1)}{(x+1)^2}$

$2y \cdot \frac{dy}{dx} = \frac{(x+1) - (x-1)}{x^2 + 2x + 1} \rightarrow \frac{dy}{dx}(2y) = \frac{2}{(x+1)^2}$

$\frac{dy}{dx} = \boxed{\frac{2}{2y(x+1)^2}}$

(5) $x = \tan y \rightarrow \frac{d}{dx}x = \frac{d}{dx}(\tan y) \rightarrow 1 = \sec^2 y \cdot \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{1}{\sec^2 y}}$

(7) $x + \tan(xy) = 0 \rightarrow \frac{d}{dx}x + \frac{d}{dx}\tan(xy) = 0$

main Rule $\rightarrow \sec^2(xy) \cdot (x\frac{d}{dx}y + y\frac{d}{dx}x) + 1 = 0 \rightarrow \sec^2(xy) \cdot (x \cdot \frac{dy}{dx} + y) + 1 = 0$

$(x \cdot \frac{dy}{dx} + y) = \frac{-1}{\sec^2(xy)} \rightarrow x \cdot \frac{dy}{dx} = -\frac{1}{\sec^2(xy)} - y \rightarrow \frac{dy}{dx} = \frac{\frac{-1}{\sec^2(xy)} - y}{x}$

(9) $x^2 + y^2 = 13, \ (-2, 3)$

$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}13 \rightarrow 2x + 2y \cdot \frac{dy}{dx} = 0$

$2(-2) + 2(3) \cdot \frac{dy}{dx} \rightarrow -4 + 6\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{4}{6}$

(11) $(x-1)^2 + (y-1)^2 = 13 \quad \frac{d}{dx}13 \quad P: (3, 4)$

$\frac{d}{dx}(x-1)^2 + \frac{d}{dx}(y-1)^2 = \rightarrow 2(x-1)(1) + 2(y-1)(1)\frac{dy}{dx} = 0$

$2(3-1) + 2(4-1)\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \boxed{-\frac{4}{6}}$

(13) $x^2y - xy^2 = 4$

$\frac{d}{dx}(x^2y) - \frac{d}{dx}(xy^2) = 0 \rightarrow (x^2\frac{d}{dx}y + y\frac{d}{dx}x^2) - (x\frac{d}{dx}y^2 + y^2\frac{d}{dx}x) = 0$

$(x^2\frac{dy}{dx} + 2xy) - (2xy\frac{dy}{dx} + y^2) = 0 \rightarrow x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 2xy$

$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ Defined for all values except $x = 0$ & $y = x/2$

15 $x^3 + y^3 = xy$

$\frac{d}{dx}(x^3) + \frac{d}{dx}y^3 = \frac{d}{dx}(xy) \rightarrow 3x^2 + 3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$

$3y^2\frac{dy}{dx} - x\frac{dy}{dx} = y - 3x^2$

$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$ Defined for all values except $y^2 = x/3$

17 $x^2 + xy - y^2 = 1,\ (2,3)$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) - \frac{d}{dx}(y^2) = 0$

3.3 HW continued

#17 $x^2+xy-y^2=1$ at $(2,3)$

$2x+x\frac{dy}{dx}+1\cdot y-2y\frac{dy}{dx}=0$

$2x+y=-x\frac{dy}{dx}+2y\frac{dy}{dx}$

$2x+y=(-x+2y)\frac{dy}{dx}$

$\frac{2x+y}{-x+2y}=\frac{dy}{dx}$

Tangent at $(2,3)$: $\frac{dy}{dx}=\frac{4+3}{-2+6}=\frac{7}{4}$

T= $y-3=\frac{7}{4}(x-2)$

N: $y-3=-\frac{4}{7}(x-2)$

#19 $x^2y^2=9$ at $(-1,3)$

$x^2\cdot 2y\frac{dy}{dx}+2xy^2=0$

$2x^2y\frac{dy}{dx}=-2xy^2$

$\frac{dy}{dx}=\frac{-2xy^2}{2x^2y}=\frac{-y}{x}$

Tangent at $(-1,3)$: $\frac{dy}{dx}=\frac{-3}{-1}=3$

T: $y-3=3(x+1)$

N: $y-3=-\frac{1}{3}(x+1)$

#23 $2xy+\pi\sin(y)=2\pi$ at $(1,\frac{\pi}{2})$

$2x\frac{dy}{dx}+2y+\pi\cos(y)\frac{dy}{dx}=0$

$2x\frac{dy}{dx}+\pi\cos(y)\frac{dy}{dx}=-2y$

$\frac{dy}{dx}(2x+\pi\cos(y))=-2y$

$\frac{dy}{dx}=\frac{-2y}{2x+\pi\cos(y)}$

Tangent at $(1,\frac{\pi}{2})$ $\frac{dy}{dx}=\frac{-2(\frac{\pi}{2})}{2+\pi\cos(\frac{\pi}{2})}=\frac{-\pi}{2+0}$

T: $y-\frac{\pi}{2}=\frac{-\pi}{2}(x-1)$

N: $y-\frac{\pi}{2}=\frac{2}{\pi}(x-1)$

#25 $y=2\sin(\pi x-y)$ at $(1,0)$

$\frac{dy}{dx}=2\cos(\pi x-y)\cdot[\pi-\frac{dy}{dx}]$

$\frac{dy}{dx}=2\pi\cos(\pi x-y)-2\cos(\pi x-y)\frac{dy}{dx}$

$\frac{dy}{dx}+2\cos(\pi x-y)\frac{dy}{dx}=2\pi\cos(\pi x-y)$

$\frac{dy}{dx}(1+2\cos(\pi x-y))=2\pi\cos(\pi x-y)$

$\frac{dy}{dx}=\frac{2\pi\cos(\pi x-y)}{1+2\cos(\pi x-y)}$

Tangent at $(1,0)$ $\frac{dy}{dx}=\frac{2\pi\cos(\pi)}{1+2\cos(\pi)}=\frac{-2\pi}{-1}=2\pi$

T: $y-0=2\pi(x-1)$

N: $y-0=\frac{-1}{2\pi}(x-1)$

replace known $\frac{dy}{dx}$

#27 $x^2+y^2=1$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{2y}=\boxed{\frac{-x}{y}}$

$\frac{dy}{dx}=\frac{-x}{y}$

$\frac{d^2y}{dx^2}=\frac{y(-1)-(-x)\frac{dy}{dx}}{y^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y}{y^2}-\frac{x^2}{y^3}=\frac{-y^2}{y^3}-\frac{x^2}{y^3}=\frac{-y^2-x^2}{y^3}=\frac{-(y^2+x^2)}{y^3}$

$=\boxed{\frac{-1}{y^3}}$

original curve: $x^2+y^2=1$

#29 $y^2=x^2+2x$

$2y\frac{dy}{dx}=2x+2$

$\frac{dy}{dx}=\frac{2x+2}{2y}=\frac{x+1}{y}$

$\frac{dy}{dx}=\frac{x+1}{y}$

$\frac{d^2y}{dx^2}=\frac{y(1)-(x+1)\frac{dy}{dx}}{y^2}=\frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}=\frac{y-\frac{(x+1)^2}{y}}{y^2}$

$\frac{d^2y}{dx^2}=\frac{y}{y^2}-\frac{(x+1)^2}{y^3}=\frac{y^2}{y^3}-\frac{(x+1)^2}{y^3}=\frac{y^2-(x+1)^2}{y^3}$

$\frac{d^2y}{dx^2}=\frac{y^2-(\overbrace{x^2+2x}^{\text{original equation}}+1)}{y^3}=\frac{y^2-(y^2+1)}{y^3}=\boxed{\frac{-1}{y^3}}$

#55 a) $x^3+y^3-9xy=0$

$3x^2+3y^2\frac{dy}{dx}-9x\frac{dy}{dx}+(-9)y=0$

$3x^2+3y^2\frac{dy}{dx}-9x\frac{dy}{dx}-9y=0$

$3y^2\frac{dy}{dx}-9x\frac{dy}{dx}=9y-3x^2$

$\frac{dy}{dx}(3y^2-9x)=9y-3x^2$

$\frac{dy}{dx}=\frac{9y-3x^2}{3y^2-9x}$

Slope at $(4,2)$: $\frac{dy}{dx}=\frac{9(2)-3(4)^2}{3(2)^2-9(4)}=\frac{18-48}{12-36}=\frac{30}{24}=\frac{5}{4}$

Slope at $(2,4)$: $\frac{dy}{dx}=\frac{9(4)-3(2)^2}{3(4)^2-9(2)}=\frac{36-12}{48-18}=\frac{4}{5}$

b) $\frac{dy}{dx}=\frac{9y-3x^2}{3y^2-9x}=0$ when $9y-3x^2=0$ so... $9y=3x^2$

Horizontal tangent when $\frac{dy}{dx}=0$ $\quad y=\frac{x^2}{3}$ plug into original!

Sub in $y=\frac{x^2}{3}$

$x^3+\left(\frac{x^2}{3}\right)^3-9x\left(\frac{x^2}{3}\right)=0$

$x^3+\frac{x^6}{27}-\frac{9x^3}{3}=0$

$x^3+\frac{x^6}{27}-3x^3=0$

$\frac{x^6}{27}-2x^3=0$

$x^3\left(\frac{x^3}{27}-2\right)=0$

$x=0$, $y=0$ origin $\quad$ or $\quad \frac{x^3}{27}-2=0$

$\frac{x^3}{27}=2$

$x^3=54$ $\quad \boxed{x=\sqrt[3]{54} \quad y=\frac{\sqrt[3]{54}^2}{3}}$

Equals: $(3\sqrt[3]{2}, 3\sqrt[3]{4})$

c) $\frac{dy}{dx} = \frac{9y-3x^2}{3y^2-9x}$ Vertical tangent when $\frac{dy}{dx}$ is undefined, so... $3y^2-9x=0$

$\frac{y^2}{3}=x$

w/ original curve:

$x^3+y^3-9xy=0$

$\left(\frac{y^2}{3}\right)^3+y^3-9\left(\frac{y^2}{3}\right)y=0$

$\frac{y^6}{27}+y^3-3y^3=0$

$\frac{y^6}{27}-2y^3=0$

$y^3\left(\frac{y^3}{27}-2\right)=0$

$\boxed{\begin{array}{c} y=0 \\ x=0 \end{array}}$ Not point A

$\frac{y^3}{27}-2=0$

$\frac{y^3}{27}=2$

$y^3=54$

$y=\sqrt[3]{54}=3\sqrt[3]{2}$ $x=\frac{\sqrt[3]{54}^2}{3}=3\sqrt[3]{4}$

$\boxed{(3\sqrt[3]{4}, 3\sqrt[3]{2})}$