Work the following on notebook paper. Write a function for each problem, and justify your answers. Give all decimal answers correct to three decimal places.

1. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

2. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum. *50 and 25*

3. A gardener wants to make a rectangular enclosure using a wall as one side and 120 m of fencing for the other three sides. Express the area in terms of x, and find the value of x that gives the greatest area.

4. A rectangle has a perimeter of 80 cm. If its width is x, express its length and area in terms of x, and find the maximum area. *Length: $40-x$ Area: $x(40-x)$ 400 ft²*

5. Suppose you had 102 m of fencing to make two side-by-side enclosures as shown. What is the maximum area that you could enclose?
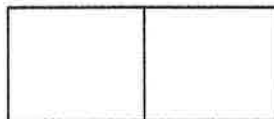
6. Suppose you had to use exactly 200 m of fencing to make either one square enclosure or two separate square enclosures of any size you wished. What plan would give you the least area? What plan would give you the greatest area?

Two squares give: 1250 m²
One square gives: 2500 m²

11. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

Problem 1 (handwritten):

$xy = 192$ $x + 3y = S$
$y = \frac{192}{x}$ Min S:
$x + 3\left(\frac{192}{x}\right) = S$
$x + \frac{576}{x} = S$
$x + 576x^{-1} = S$
$1 - 576x^{-2} = S'$
$1 - 576x^{-2} = 0$
$1 = \frac{576}{x^2}$
$x^2 = 576$
$x = \pm 24$ if $x = 24$ $y = 8$

Problem 3 (handwritten):

WALL, y, x
$2x + y = 120$ $xy = A$
$y = 120 - 2x$ Max A:
$A = x(120 - 2x)$
$A = 120x - 2x^2$
$A' = 120 - 4x = 0$
$x = 30$ ft

Problem 5 (handwritten):

x, x, y, y, y
$4x + 3y = 102$ $2xy = A$
Max A: $3y = 102 - 4x$
$y = 34 - \frac{4}{3}x$
$2x(34 - \frac{4}{3}x) = A$
$68x - \frac{8x^2}{3} = A$
$A' = 68 - \frac{16x}{3}$
$\frac{16x}{3} = 68$
$x = 12.75$ $y = 17$
Max area: 433.5 m²

Problem 11 (handwritten):

$108 = x^2 + 4xy$
$V = x^2 y$
Max V:
$108 = x^2 + 4xy$ $V = x^2\left(\frac{108 - x^2}{4x}\right)$
$108 - x^2 = 4xy$
$\frac{108 - x^2}{4x} = y$ $V = \frac{108x^2 - x^4}{4x}$
$V = 27x - \frac{1}{4}x^3$
$V' = 27 - \frac{3}{4}x^2$
$27 = \frac{3}{4}x^2$ If $x = 6, y = 3$
$36 = x^2$ $x = \pm 6$ $6 \times 6 \times 3$