Name________________________________

Optimization Problems Practice

Date____________________ Period____

Solve each optimization problem.

1) A company has started selling a new type of smartphone at the price of $\$110 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$50 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit? (Remember that Profit = Revenue - Cost)

Optimize: max profit
How many phones

Equations: $P = r - c$

$P = x(110 - 0.05x) - (50x + 6000)$

$P = x(110 - 0.05x) - 50x - 6000$

$P = 110x - 0.05x^2 - 50x - 6000$

$P = -0.05x^2 + 60x - 6000$

$P' = -0.10x + 60$

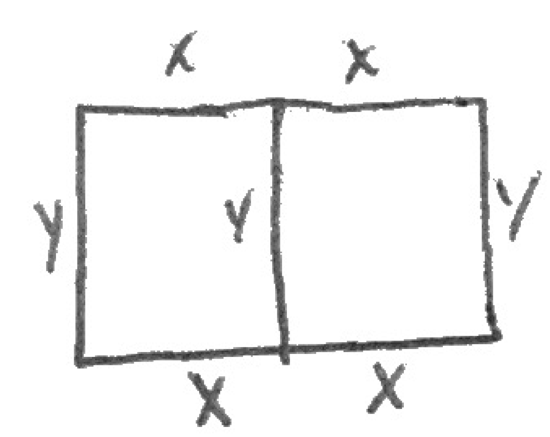
$0 = -0.10x + 60$

$-60 = -0.10x$

$\frac{-60}{-0.10} = x$

$x = 600$ phones

2) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?



Optimize: max area
What dimensions

Equations: $2x \cdot y = A$

$4x + 3y = 200$

$3y = 200 - 4x$

$y = \frac{200 - 4x}{3}$

$2x\left(\frac{200 - 4x}{3}\right) = A$

$\frac{400x - 8x^2}{3} = A'$

$\frac{400}{3} - \frac{16}{3}x = A'$

$\frac{400}{3} - \frac{16x}{3} = 0$

$400 - 16x = 0$

$x = 25$ ft

$y = \frac{100}{3}$ ft

3) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 8 greater than the negative number. What final product is the expert looking for?

Optimize: min product

Equations: x and y x is + y is negative

$xy = A$

$x = y + 8$

$x(y) = A$

$(y + 8)(y) = A$

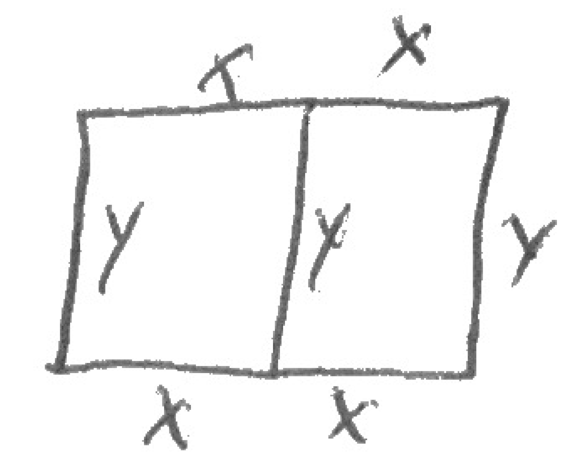
$y^2 + 8y = A$

$2y + 8 = A'$

$2y + 8 = 0$

$y = -4$ $x = 4$ Product: -16

4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?



Optimize max area: dimensions

Equations

$4x+3y=400$

$2xy=A$

$3y=400-4x$

$y=\frac{400-4x}{3}$

$2x\left[\frac{400-4x}{3}\right]=A$

$\frac{800x-8x^2}{3}=A$

$\frac{800}{3}-\frac{16}{3}x=A'$

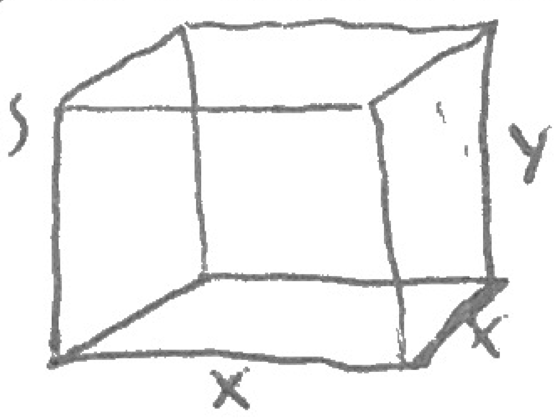
$\frac{800}{3}-\frac{16x}{3}=0$

$800-16x=0$

$x=50ft \quad y=\frac{200}{3} ft$

5) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

Optimize: minimum glass (surface area): dimensions



Equations $x^2y=500$

$y=\frac{500}{x^2}$

☆ $x^2+4xy=S$

$x^2+\frac{4x\cdot 500}{x^2}=S$

$x^2+\frac{2000x}{x^2}=S$

$x^2+\frac{2000}{x}=S$

$x^2+2000x^{-1}=S$

$2x-2000x^{-2}=S$

$2x-\frac{2000}{x^2}=S'$

$-\frac{2000}{x^2}=-2x$

$-2000=-2x^3$

$1000=x^3$

$x=10 ft$

$y=5 ft$

$\boxed{10\times10\times5}$

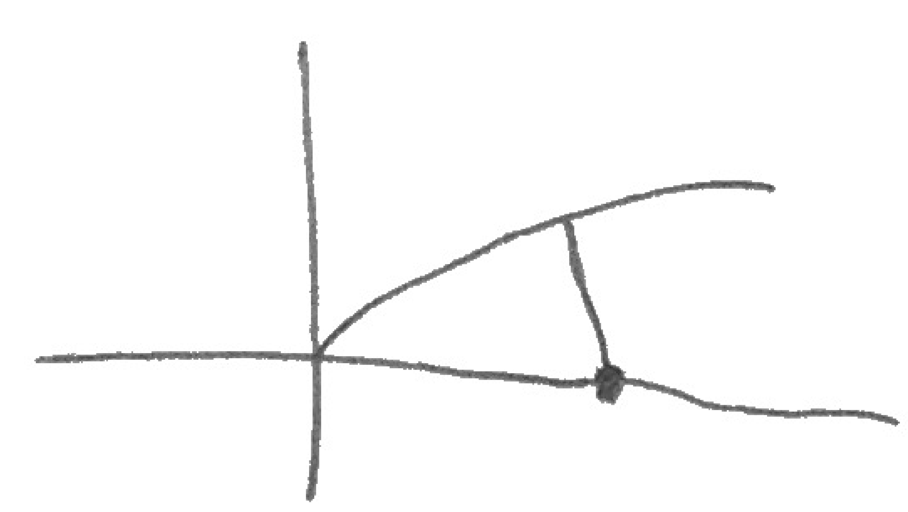
6) Which point on the graph of $y=\sqrt{x}$ is closest to the point (5, 0)?

Optimize: distance between $y=\sqrt{x}$ and (5,0) is min

Equation:

Distance formula:

☆ $d=\sqrt{(x-5)^2+(y-0)^2}$



$y=\sqrt{x}$

$d=\sqrt{(x-5)^2+(\sqrt{x})^2}$

$d=\sqrt{x^2-10x+25+x}$

$d=\sqrt{x^2-9x+25}$

$d=(x^2-9x+25)^{1/2}$

$d'=\frac{1}{2}(x^2-9x+25)^{-1/2}(2x-9)$

$d'=\frac{1}{2\sqrt{x^2-9x+25}}(2x-9)$

CP $f'=0$ OR DNE

$2\sqrt{x^2-9x+25}=0$

$\sqrt{x^2-9x+25}=0$

$x^2-9x+25=0$

never

$2x-9=0$

$x=\frac{9}{2}$

$y=\frac{3\sqrt{2}}{2}$

$\left(\frac{9}{2}, \frac{3\sqrt{2}}{2}\right)$