Homework

For each function F(x), find F'(x).

1) $F(x) = \int_{-4}^{x} (t-1)\,dt$

$F'(x) = \frac{d}{dx}\int_{-4}^{x}(t-1)dt = x-1$

2) $F(x) = \int_{-3}^{x} (t^2+2t+3)\,dt$

$F'(x) = \frac{d}{dx}\int_{-3}^{x} t^2+2t+3\,dt =$
x^2+2x+3

end is not just x

chain rule!

5) $F(x) = \int_{2}^{x^3} \frac{1}{t^3}\,dt$

$F'(x) = \frac{1}{(x^3)^3}\cdot(3x^2) = \frac{3x^2}{x^9} = \frac{3}{x^7}$

don't forget parentheses

6) $F(x) = \int_{x}^{x^2} (-2t-2)\,dt$

$F'(x) = (-2(x^2)-2)(2x) - (-2x-2)$
$= (-2x^2-2)(2x) + 2x + 2$
$= -4x^3 - 4x + 2x + 2 = -4x^3 - 2x + 2$

7) $F(x) = \int_{x}^{x^2} (t^2-8t+11)\,dt$

$F'(x) = ((x^2)^2 - 8(x^2)+11)(2x) - (x^2-8x+11)$
$= (x^4-8x^2+11)(2x) - x^2+8x-11 = 2x^5 - 16x^3 + 22x - x^2 + 8x - 11$
$= 2x^5 - 16x^3 - x^2 + 30x - 11$

8) $F(x) = \int_{x}^{2x} \frac{2}{t}\,dt$

$F'(x) = \left(\frac{2}{2x}\right)\cdot 2 - \left(\frac{2}{x}\right)$
$= \frac{4}{2x} - \frac{2}{x} = \frac{2}{x} - \frac{2}{x} = 0$

What if $M(x) = \int_{1}^{\sin(x)} (2t+6t^2)dt$ Determine M'(x) by writing an equation for M(x) without the integrand, then taking its derivative.

$M(x) = (t^2+2t^3)\Big|_{1}^{\sin(x)} = \left[(\sin(x))^2 + 2(\sin(x))^3\right] - \left[(1)^2 + 2(1)^3\right]$

$\frac{d}{dx}$ is: $2\sin(x)\cdot\cos(x) + 6(\sin(x))^2\cdot\cos(x) - 0$
$\left[2\sin(x) + 6(\sin(x))^2\right]\cos(x)$

In general : $\frac{d}{dx}\int_{a}^{g(x)} f(t)dt =$ $f(g(x))\cdot g'(x)$

11. $\frac{d}{dx}[\int_{\pi}^{2x} \frac{t}{1+t^2} dt]$

$\frac{2x}{1+(2x)^2} \cdot (2) = \frac{4x}{1+4x^2}$

12. $\frac{d}{dx}[\int_{e^x}^{3} arcsin(t)dt]$

$-\arcsin(e^x) \cdot e^x$

13. If $G(x) = \int_0^{3x} \frac{t}{1+t^2} dt$, determine G'(2)

$G'(x) = \frac{3x}{1+(3x)^2} (3) = \frac{9x}{1+9x^2}$

$G'(2) = \frac{18}{37}$

14. If $\mathrm{H(x)} = \int_{2x}^{5} (t^2 + 3t)dt$, determine H'(1)

$H'(x) = -((2x)^2 + 3(2x)) \cdot (2)$

$= -(4+6)2 = -20$

15. $\frac{d}{dx}[\int_x^{x^3} \cos(t)\, dt] = \cos(x^3) \cdot 3x^2 - \cos(x)$

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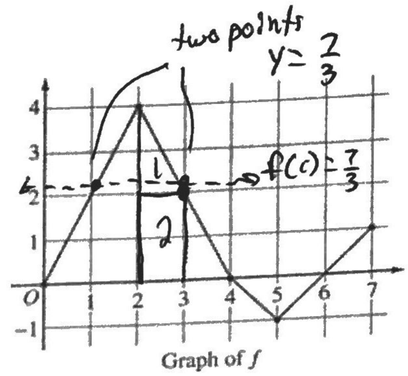
Question 5



Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

$g(x)$ area
$g'(x)=f(x)$ y-value
$g''(x)=f'(x)$ slope

a) $g(3)=\int_2^3 f(t)\,dt = 3$ $\quad g'(3)=f(3)=2$ $\quad g''(3)=f'(3)$ slope at $x=3$ $\quad -2$

b) This is tricky; we want average rate of change of $\underline{g}$. Not f.

So we need $g(0)=\int_2^0 f(t)\,dt = -4$ $\quad (0,-4)$

$g(3)=\int_2^3 f(t)\,dt = 3$ $\quad (3,3)$

$\frac{3-(-4)}{3-0} = \frac{7}{3}$

c) $g'(c)$ is $f(c)$

So, how many times does $f(c)=\frac{7}{3}$?

Two times, because graph of f intersects $y=\frac{7}{3}$ twice on $(0,3)$

d) POI of g is when $g''(x)=f'(x)$ changes sign.

Look for relative extrema of f.

$x=2$ and $x=5$

You could also say, f changes from increasing to decreasing or decreasing to increasing at those x-values.