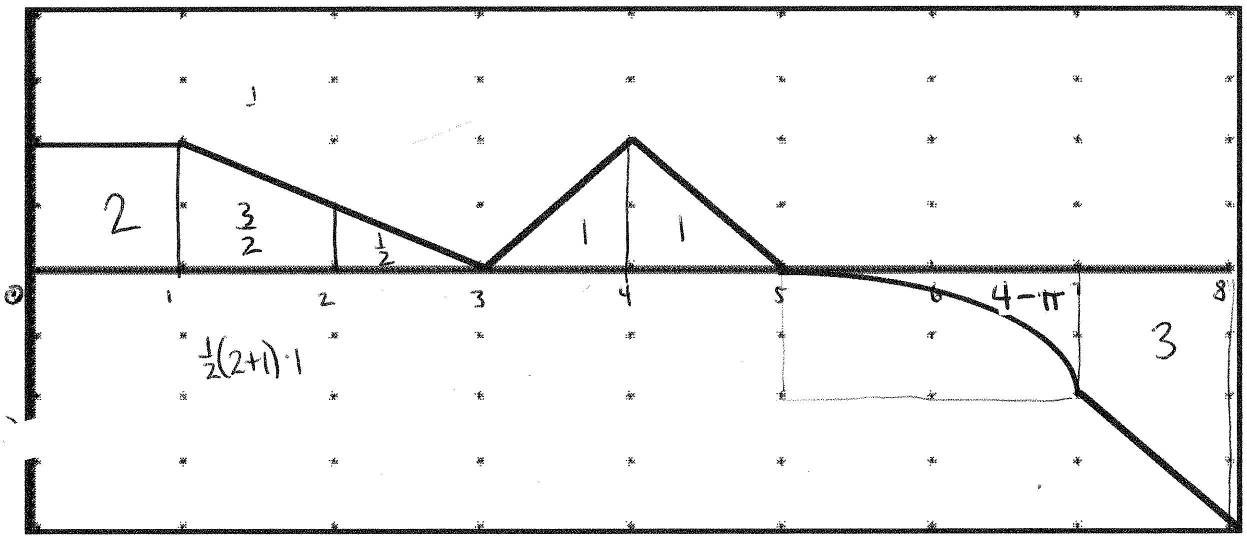
The Accumulation Function - Homework



$y = f(x)$

$\frac{1}{2}(2+4)1$

$4 - \frac{\pi 2^2}{4} = 4-\pi$

1. Let $F(x) = \int_0^x f(t)\,dt$ where the graph of $f(x)$ is above (the graph consists of lines and a quarter circle)

$6-(4-\pi)=2+\pi$ $\quad 2+\pi-3$

a. Complete the chart

x	0	1	2	3	4	5	7	8
$F(x)$	0	2	$\frac{7}{2}$	4	5	6	$2+\pi$	$-1+\pi = \pi-1$
$F'(x)$	2	2	1	0	2	0	-2	-4

b. On what subintervals of [0, 8] is F increasing? Decreasing? Justify your answer.

(0, 5) F increasing since $F'(x)=f(x)$ is positive (5, 8) F decreasing since $F'(x)=f(x)$ is negative

c. Where in the interval [0, 8] does F achieve its minimum value? What is the minimum value? Justify answer.

abs min at $x=0$ compare values at end points @ critical pts (where $F'(x)=f(x)=0$).
$\boxed{F(0)=0}$, $F(3)=4$, $F(5)=6$, $F(8)=\pi-1$

d. Where in the interval [0, 8] does F achieve its maximum value? What is the maximum value? Justify answer.

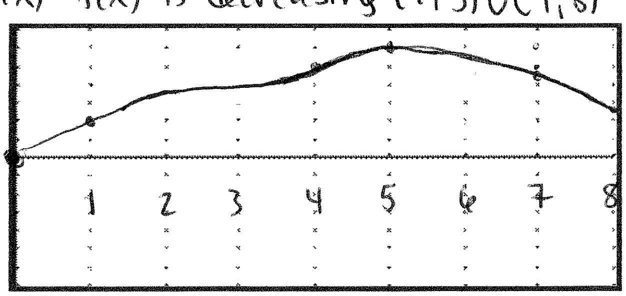
$\boxed{\text{abs max at } x=5.\ F(5)=6}$. Closed interval test (compare values of $F(x)$ at endpts and critical points.

e. Find the concavity of F and any inflection points. Justify answers.

$F(x)$ concave up when $F'(x)=f(x)$ is increasing (3,4).
$F(x)$ concave down when $F'(x)=f(x)$ is decreasing $(1,3)\cup(4,8)$

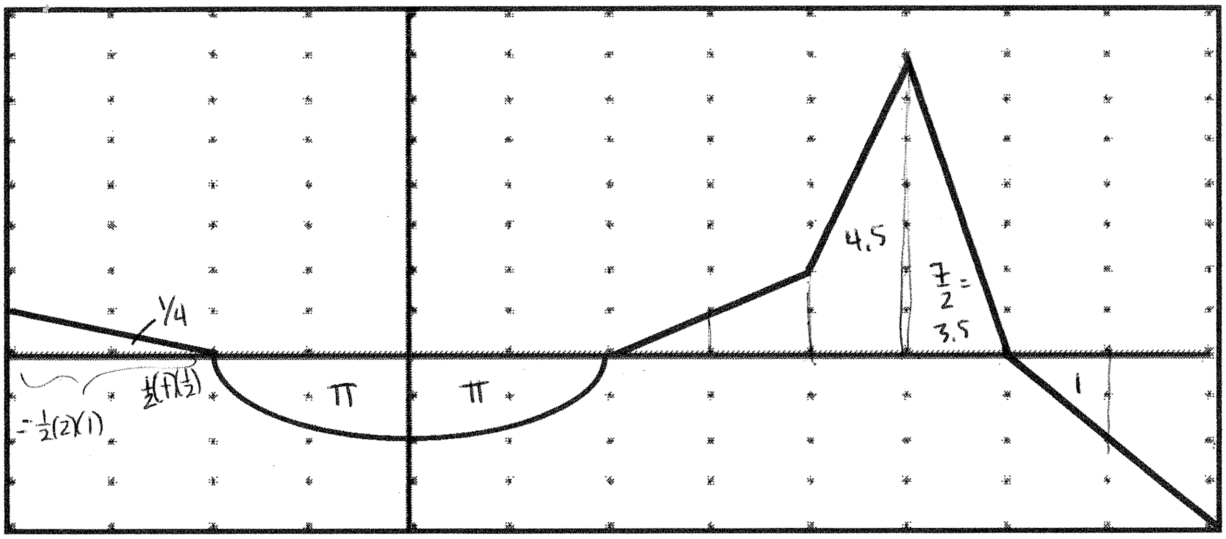
pts of inflection: (3,4) (4,5) x values where $F(x)$ Δ's concavity & $y=F(x)$ value.

f. Sketch a rough graph of $F(x)$



Stu Schwartz

$\frac{\pi (2)^2}{2}$ $\frac{1}{2}(2+7)1 = \frac{9}{2} = 4.5$



$F(-2) = \int_0^{-2} f(t)dt$

$F(-2) = -\int_{-2}^{0} f(t)dt$
$= - -\pi = \pi$

$y = f(x)$

$F(-3) = -\int_{-3}^{0} f(t)dt$
$= -(\frac{1}{4} - \pi)$

$f(x)$

2. Let $F(x) = \int_0^x f(t)\,dt$ where the graph of $f(x)$ is above (the graph consists of lines and a semi-circle)

a. Complete the chart

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$F(x)$	$\pi - 1$	$\pi - \frac{1}{4}$	π	$\approx \frac{2\pi}{3}$	0	$\approx \frac{-2\pi}{3}$	$-\pi$	$\frac{1}{2} - \pi$	$2 - \pi$	$6.5 - \pi$	$10 - \pi$	$9 - \pi$	$6 - \pi$
$F'(x)$	1	0.5	0	≈ -1.8	-2	≈ -1.8	0	1	2	7	0	-2	-4

b. On what subintervals of $[-4,8]$ is F increasing? Decreasing?

$F(x)$ increasing when $F'(x) = f(x)$ positive $(-4,-2) \cup (2,6)$; $F(x)$ decreasing when $F'(x) = f(x)$ negative: $(-2,2) \cup (6,8)$

c. Where in the interval $[-4,8]$ does F achieve its minimum value? What is the minimum value? Justify answer.

$F(x)$ has min at $x = 2$ $\boxed{F(2) = -\pi}$ check end pts and where $F'(x) = f(x)$ changes from negative to positive

X	F(X)
-4	$\pi - 1$
2	$-\pi$
8	$6 - \pi$

d. Where in the interval $[-4,8]$ does F achieve its maximum value? What is the maximum value? Justify answer.

$F(x)$ has max at $x = 6$ $\boxed{F(6) = 10 - \pi}$ Check endpoints and where $F'(x) = f(x)$ changes from positive to negative

x	F(X)
-4	$\pi - 1$
-2	π
$\boxed{6}$	$\boxed{10 - \pi}$
8	-4

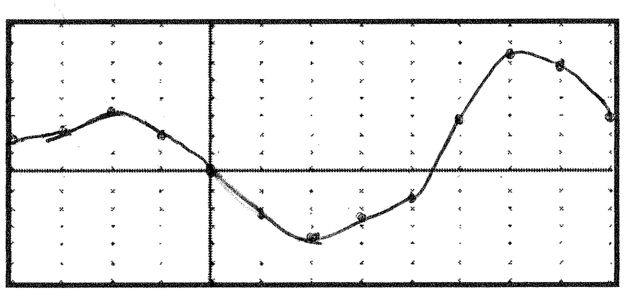
e. On what subintervals of $[-4,8]$ is F concave up and concave down? Find its inflection points. Justify answers.

F concave up when $F'(x) = f(x)$ increasing $(0,5)$

F concave down when $F'(x) = f(x)$ decreasing: $(-4,0) \cup (5,8)$

pt of inflection
$(0, F(0)) = (0,0)$
$(5, F(5)) = (5, 6.5 - \pi)$

f. Sketch a rough graph of $F(x)$



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