Homework

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.
   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

a) $\Delta t = \frac{24-0}{4} = 6$

$\int_0^{24} R(t)dt \approx 6(10.4)+6(11.2)+6(11.3)+6(10.2) = 258.6$ gallons

This approximates the amount of water in gallons that flowed out of the pipe from 0-24hrs.

b) Asked about $R'(t)$ on $(0,24)$. R is differentiable, so the MVT can be applied.
$(0, 9.6)$ and $(24, 9.6)$

$\frac{9.6-9.6}{24-0} = \frac{0}{24} = 0$ → by the MVT, there must be a t on $0<t<24$ where $R'(t)=0$.

Question 4

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

a) point: $(1,15)$
slope: $f'(1) = 8$

$y-15 = 8(x-1)$
$y = 8(x-1)+15$
at $x=1.4$ $y = 8(1.4-1)+15 = = 18.2$

b) $\int_1^{1.4} f'(x)dx \approx 0.2(10)+0.2(13) = 4.6$

$\Delta x = 0.2$

$f(1.4) \approx f(1) + \int_1^{1.4} f'(x)dx$
$\approx 15 + 4.6 = 19.6$

we're not learned this yet, technically. But it is like Ex1 in the lesson.

AP Calc MRAM Homework

1.

intervals:	[0,2]	[2,4]	[4,6]	[6,8]	[8,10]
midpoint:	1	3	5	7	9
height:	12	10	13	6	6
area:	$2\cdot 12$	$2\cdot 10$	$2\cdot 13$	$2\cdot 6$	$2\cdot 6$

Total area $= 2[12+10+13+6+6] = 2[47] = \boxed{94 \text{ inches}}$
(Total distance traveled)

2.

intervals:	[0,20]	[20,40]	[40,60]	[60,80]	[80,100]
midpoint:	10	30	50	70	90
height:	40	35	32	20	10
area:	20(40)	20(35)	20(32)	20(20)	20(10)

Total area = Total gallons $= 20[40+35+32+20+10] = \boxed{2740 \text{ gallons}}$
added to tank in 100 min

3. length: $\frac{1 \text{ hr}}{6 \text{ intervals}} = \frac{1}{6}$ hr

intervals:	$[0, \frac{1}{6}]$,	$[\frac{1}{6}, \frac{2}{6}]$,	$[\frac{2}{6}, \frac{3}{6}]$,	$[\frac{3}{6}, \frac{4}{6}]$	$[\frac{4}{6}, \frac{5}{6}]$	$[\frac{5}{6}, \frac{6}{6}]$
midpoint:	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{9}{12}$	$\frac{11}{12}$
height $v(m_i)$	30	45	60	60	45	45
area:	$\frac{1}{6}(30)$	$\frac{1}{6}(45)$	$\frac{1}{6}(60)$	$\frac{1}{6}(60)$	$\frac{1}{6}(45)$	$\frac{1}{6}(45)$

Total area = total distance $= \frac{1}{6}(30+45+60+60+45+45) = 47.5$ miles
traveled from 2-3 pm

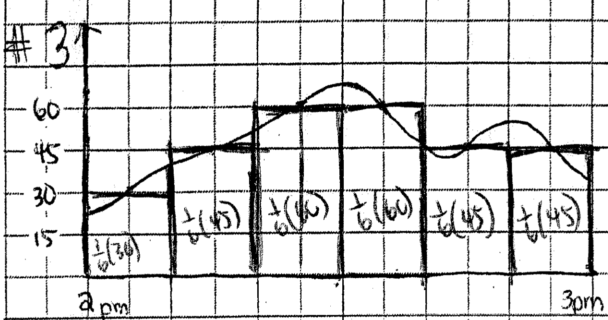
Total distance traveled all day $= 65 + 47.5 = \boxed{112.5 \text{ miles}}$

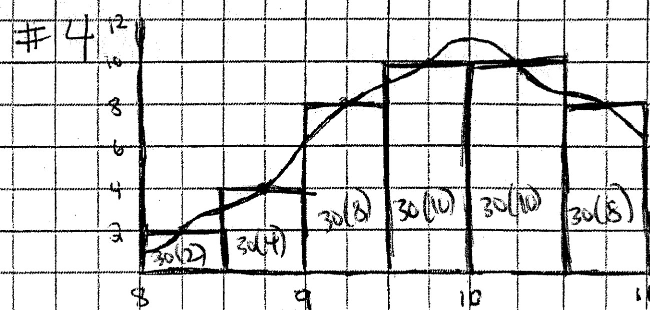
4. length $= \frac{11-8}{6} = \frac{3}{6} = \frac{1}{2}$ hr $= 30$ min

intervals:	[8, 8:30]	[8:30, 9]	[9, 9:30]	[9:30, 10]	[10, 10:30]	[10:30-11]
midpt:	8:15	8:45	9:15	9:45	10:15	10:45
height: $v(m_i)$	2	4	8	10	10	8
area:	30(2)	30(4)	30(8)	30(10)	30(10)	30(8)

Total area = total # of ppl : $30(2+4+8+10+10+8) = 30\text{min}(42 \frac{\text{ppl}}{\text{min}}) = \boxed{1260 \text{ ppl}}$
entered in 3 hrs

MRAM HWK graphs





6.1 HW

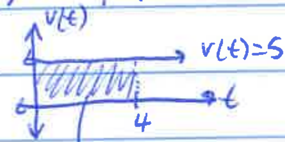
P. 277 #1–9 odd, 17–18   P. 320 #1–7, 31, 32, 34

1. Starting position: 0 inother words $P(0)=0$; $P(t)$ measures position

$P(4)=0+\int_0^4 v(t)\,dt$

$P(4)=0+\int_0^4 5\,dt$

$P(4)=0+20=20$



$4(5)=20$

Basic rate × time

3. Rate: 3 yd^3/day

$3(365)=1095\ yd^3$

This is $\int_0^{365} 3\,dt$

5. $P(0)=0$ → starting position

$v(t)=t^2+1$

$P(4)=0+\int_0^4 t^2+1\,dt$

Midpoint! four Rect

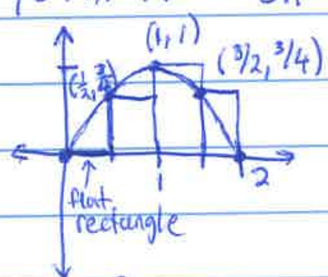
$\Delta t=\frac{4-0}{4}=1$

t	0	1/2	1	3/2	2	5/2	3	7/2	4
$v(t)$	1	1.25	2	3.25	5	7.25	10	13.25	17

$\int_0^4 t^2+1\,dt \approx 1(1.25)+1(3.25)+1(7.25)+1(13.25)=25$

$P(4)=0+25=25$

7. a) $y=2x-x^2$ on $0\le x\le 2$



x	y
0	0
1/2	0.75
1	1
3/2	0.75
2	0

$\Delta x=\frac{2-0}{4}=\frac{1}{2}$

$\int_0^2 2x-x^2\,dx \approx \frac{1}{2}(0)+\frac{1}{2}(3/4)+\frac{1}{2}(1)+\frac{1}{2}(3/4)=5/4$

9.

n	$LRAM_n$	$MRAM_n$	$RRAM_n$
10	1.32	1.34	1.32
50	1.3328	1.3336	1.3328
100	1.3332	1.3334	1.3332
500	1.333328	1.333336	1.333329

see page 275

17. Cardiac output = $\frac{\text{mg of dye}}{\text{units of area under curve}}$

mg of dye = 5 mg we need $\int_2^{24}$ Dye Concentration dt

I'll use Left Riemann Sum

$2(0+0.6+1.4+2.7+3.7+4.1+3.8+2.9+1.7+1.0+0.5) = 44.8$

Cardiac Output = $\frac{5}{44.8} = 0.1116$ L/sec $\cdot \frac{60 \text{ sec}}{\text{min}} = 6.696$ L/min

18. a) LRAM

$\Delta t = \frac{10-0}{10} = 1$ sec $1(0+12+22+10+5+13+11+6+2+6) = 87$ inches

sec · in/sec = in

b) RRAM $1(12+22+10+5+13+11+6+2+6+0) = 87$ inches

P. 320 1. $\int_0^2 x\,dx$

a)

x	0	1/2	1	3/2	2
y	0	1/2	1	3/2	2

TRAP: $\frac{(0+1/2)1/2}{2} + \frac{(1/2+1)1/2}{2} + \frac{(1+3/2)1/2}{2} + \frac{(3/2+2)1/2}{2}$

$= 2$

b) $y = x$ $y' = 1$ $y'' = 0$ so TRAP is a perfect approximation

c) skip

2. $\int_0^2 x^2\,dx$

a)

x	0	1/2	1	3/2	2
y	0	1/4	1	9/4	4

TRAP: $\frac{(0+1/4)1/2}{2} + \frac{(1/4+1)1/2}{2} + \frac{(1+9/4)1/2}{2} + \frac{(9/4+4)1/2}{2}$

b) $y = x^2$ $y' = 2x$ $y'' = 2$ so $y = x^2$ is CCU. My trapezoidal approximation is an over approximation.

c) skip

3. $\int_0^2 x^3 dx$

a)

x	0	1/2	1	3/2	2
y	0	1/8	1	27/8	8

TRAP: $\frac{(0+1/8)\,1/2}{2} + \frac{(1/8+1)\,1/2}{2} + \frac{(1+27/8)\,1/2}{2} + \frac{(27/8+8)\,1/2}{2}$

b) $y=x^3$ $y'=3x^2$ $y''=6x$ on $(0,2)$ $6x$ is positive, so $y=x^3$ is ccu.
Thus, my approximation with trapezoids is an over approximation.

c) skip

4. $\int_1^2 \frac{1}{x} dx$

a)

x	1	5/4	3/2	7/4	2
y	1	4/5	2/3	4/7	1/2

TRAP $= \frac{(1+4/5)\,1/4}{2} + \frac{(4/5+2/3)\,1/4}{2} + \frac{(2/3+4/7)\,1/4}{2} + \frac{(4/7+1/2)\,1/4}{2}$

b) $y=x^{-1}$ $y'=-x^{-2}$ $y''=2x^{-3}=\frac{2}{x^3}$. On $(1,2)$ $\frac{2}{x^3}$ is positive. Thus $y=\frac{1}{x}$ is
CCU and my trapezoidal approximation is an over approximation.

c) skip

5. $\int_0^4 \sqrt{x}\, dx$

a)

x	0	1	2	3	4
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2

TRAP: $\frac{(0+1)1}{2} + \frac{(1+\sqrt{2})1}{2} + \frac{(\sqrt{2}+\sqrt{3})1}{2} + \frac{(\sqrt{3}+2)1}{2}$

b) $y=x^{1/2}$ $y'=\frac{1}{2}x^{-1/2}$ $y''=-\frac{1}{4}x^{-3/2}=\frac{-1}{4\sqrt{x^3}}$. On $(0,4)$, $\frac{-1}{4\sqrt{x^3}}$ is negative. Thus $y=\sqrt{x}$
is CCD and my Trapezoidal approximation is an under approximation

c) skip

6. $\int_0^\pi \sin(x) dx$

a)

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
y	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0

TRAP: $\frac{(0+\sqrt{2}/2)\,\pi/4}{2} + \frac{(\sqrt{2}/2+1)\,\pi/4}{2} + \frac{(1+\sqrt{2}/2)\,\pi/4}{2} + \frac{(\sqrt{2}/2+0)\,\pi/4}{2}$

b) $y=\sin(x)$ $y'=\cos(x)$ $y''=-\sin(x)$. On $(0,\pi)$, $-\sin(x)$ is negative. Thus, $y=\sin(x)$
is CCD and my trapezoidal approximation is an underapproximation

7. $\int_0^6 f(x)dx \approx \frac{(12+10)1}{2} + \frac{(10+9)1}{2} + \frac{(9+11)1}{2} + \frac{(11+13)1}{2} + \frac{(13+16)1}{2} + \frac{(16+18)1}{2} = 74$

31. False the Trapezoidal Rule will overestimate $\int_a^b f(x)dx$ if f is CCU on $[a,b]$ because the top of each trapezoid will lie above f.

32. Generally True for small n values b/c the Trapezoidal Rule averages LRAM and RRAM. But as $n \to \infty$ this becomes $\boxed{\text{False}}$! Eventually the width of each trapezoid or rectangle gets so small, the difference in approximations goes to 0.

34. $\int_{-2}^{4} \frac{e^x}{2} dx$

$\Delta x = \frac{4+2}{3} = 2$

x	-2	0	2	4
y	$\frac{e^{-2}}{2}$	$\frac{1}{2}$	$\frac{e^2}{2}$	$\frac{e^4}{2}$

TRAP: $\frac{\left(\frac{e^{-2}}{2}+\frac{1}{2}\right)2}{2} + \frac{\left(\frac{1}{2}+\frac{e^2}{2}\right)2}{2} + \frac{\left(\frac{e^2}{2}+\frac{e^4}{2}\right)2}{2} = \frac{e^{-2}}{2}+\frac{1}{2}+\frac{1}{2}+\frac{e^2}{2}+\frac{e^2}{2}+\frac{e^4}{2}$

$= \frac{1}{2}(e^{-2}+1+1+e^2+e^2+e^4)$

$= \frac{1}{2}(e^{-2}+2+2e^2+e^4)$

D