6.2

Homework

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

Calc OK

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

deriv.

a) $\int_0^{40} v(t)dt \approx 10(9.2+7.0+2.4+4.3) = 229$ miles $\int_0^{40} v(t)dt$ gives the distance in miles traveled by the plane from 0–40 minutes.

b) acceleration is the derivative (slope) of v(t). v(t) is differentiable. So the MVT can be applied.
(0, 7.0) and (15, 7.0) (25, 2.4) and (30, 2.4). So by the MVT, there must be at least 2 instances on $0<t<40$ where $a(t)=0$, on (0, 15) and (25, 30)

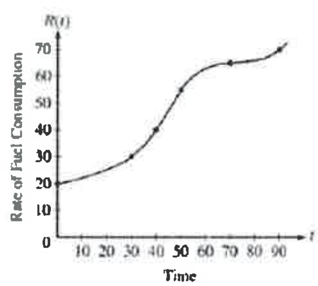
$\frac{7.0-7.0}{15-0} = 0$ and $\frac{2.4-2.4}{30-25} = 0$

c) $f(t) = 6 + \cos(\frac{t}{10}) + 3\sin(\frac{7t}{40})$ → $f'(23) = -0.407$ or -0.408 mi/min^2
could use nDeriv on calculator.

chain Rule $f'(t) = -\sin(\frac{t}{10})\frac{1}{10} + 3\cos(\frac{7t}{40})\frac{7}{40}$

Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption (R(t)) is increasing fastest (R'(t)) at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

a) $R'(45) \approx \frac{R(50)-R(40)}{50-40} = \frac{55-40 \text{ gal/min}}{10 \text{ min}} = \frac{15}{10}$ gal/min^2 $= 1.5$ gal/min^2

b) $R'(t)$ has a max at $t=45$ min. If $R'(t)$ has a max and $R(t)$ is twice-differentiable, $R''(45)=0$.

c) $\int_0^{90} R(t)dt \approx 30(20)+10(30)+10(40)+20(55)+20(65) = 3700$ gal
This approximation is an underapproximation b/c $R(t)$ is increasing and I used a Left Riemann Sum.

In the chart below, a definite integral has been provided. Please write the limit of the Riemann Sum that corresponds with the definite integral.

Definite Integral	Limit of Riemann Sum
$\int_{-2}^{3} \sqrt{x+4}dx$	$\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\left(-2+\frac{5k}{n}\right)+4}\ \frac{5}{n}$
$\int_{-2}^{3} \sin(x)x^2\,dx$	$\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(-2+\frac{5k}{n}\right)\left(-2+\frac{5k}{n}\right)\frac{5}{n}$
$\int_{4}^{6} 3x - 2\ln(x)\,dx$	$\lim_{n\to\infty} \sum_{k=1}^{n} 3\left(4+\frac{2k}{n}\right)-2\ln\left(4+\frac{2k}{n}\right)\frac{2}{n}$
$\int_{2}^{8} \frac{1}{x}dx$	$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2+\frac{6k}{n}}\ \frac{6}{n}$

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#1. $\int_0^2 x^2 dx$

#2. $\int_{-7}^{5} (x^2-3x)\,dx$

#3. $\int_1^4 \frac{1}{x}dx$

#4 $\int_2^3 \frac{1}{1-x}dx$

#5 $\int_0^1 \sqrt{4-x^2}\,dx$

#6 $\int_{-\pi}^{\pi} \sin^3(x)\,dx$

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