The Fundamental Theorem of Calculus - Homework

Find the value of the definite integrals below.

1. $\int_0^1 3x\,dx = \frac{3x^2}{2}\Big]_0^1 =$

$\frac{3}{2} - 0 = \boxed{\frac{3}{2}}$

2. $\int_{-2}^{3} (x-5)\,dx = \frac{x^2}{2} - 5x\Big]_{-2}^{3}$

$\left(\frac{9}{2} - 15\right) - \left(\frac{4}{2} + 10\right) = \frac{5}{2} - 25$

$= \boxed{-\frac{45}{2}}$

3. $\int_{-1}^{4} (x^2 + 2x - 1)\,dx = \frac{x^3}{3} + \frac{2x^2}{2} - x\Big]_{-1}^{4}$

$= \left(\frac{64}{3} + 16 - 4\right) - \left(\frac{-1}{3} + 1 + 1\right) =$

$= \frac{65}{3} + 10 = \boxed{\frac{95}{3}}$

4. $\int_0^2 (2x-5)^2\,dx = \int (4x^2 - 20x + 25)\,dx$

$\frac{4x^3}{3} - \frac{20x^2}{2} + 25x\Big]_0^2 = \left(\frac{4\cdot 8}{3} - 10(4) + 50\right) - (0) =$

$\frac{32}{3} - 40 + 50 = \boxed{\frac{62}{3}}$

$\frac{32}{3} + \frac{30}{3}$

5. $\int_2^3 \left(\frac{4}{x^2} + 1\right) dx = \int_2^3 4x^{-2} + 1\,dx$

$\frac{4x^{-1}}{-1} + x = \frac{-4}{x} + x\Big]_2^3 =$

$\left(-\frac{4}{3} + 3\right) - \left(-\frac{4}{2} + 2\right) = \frac{-4}{3} + 3 = \boxed{\frac{5}{3}}$

6. $\int_{-2}^{-1} \left(x - \frac{1}{x^2}\right) dx = \int_{-2}^{-1} (x - x^{-2})\,dx =$

$\frac{x^2}{2} + x^{-1} = \frac{x^2}{2} + \frac{1}{x}\Big]_{-2}^{-1} = \left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} - \frac{1}{2}\right)$

$-\frac{1}{2} - \frac{3}{2} = \boxed{-2}$

7. $\int_1^9 \frac{x-2}{\sqrt{x}}\,dx = \int_1^9 \frac{x}{x^{1/2}} - \frac{2}{x^{1/2}}\,dx =$

$\int_1^9 x^{1/2} - 2x^{-1/2}\,dx = \frac{2x^{3/2}}{3} - 2\cdot 2x^{1/2}\Big]_1^9 =$

$\left(\frac{2}{3}(9^{3/2}) - 4(9^{1/2})\right) - \left(\frac{2}{3} - 4\right) = \frac{2}{3}(27) - 12 - \frac{2}{3} + 4$

$= 18 - 12 - \frac{2}{3} + 4 = 10 - \frac{2}{3} = \boxed{\frac{28}{3}}$

8. $\int_{-2}^{2} \sqrt[3]{x}\,dx = \int_{-2}^{2} x^{1/3}\,dx =$

$\frac{3x^{4/3}}{4}\Big]_{-2}^{2} = \frac{3}{4}\left(2^{4/3} - (-2)^{4/3}\right)$

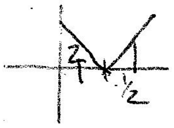
$= \frac{3}{4}\left(2^{4/3} - 2^{4/3}\right)$

$\boxed{= 0}$

9. $\int_0^1 \left(t^{2/3} - t^{1/3}\right) dt = \frac{3t^{5/3}}{5} - \frac{3t^{4/3}}{4}\Big]_0^1$

$= \frac{3}{5} - \frac{3}{4} - 0 = \frac{12}{20} - \frac{15}{20} = \boxed{\frac{-3}{20}}$

10. $\int_0^3 |x-2|\,dx = \boxed{2.5}$



11. $\int_{-\pi/2}^{\pi/2} \cos x\,dx = \sin x\Big]_{-\pi/2}^{\pi/2}$

$= \sin(\pi/2) - \sin(-\pi/2)$

$= 1 - -1 = \boxed{2}$

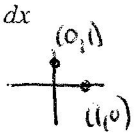
12. $\int_0^{\pi} (2x - \sin x)\,dx = \frac{2x^2}{2} + \cos x\Big]_0^{\pi}$

$= \pi^2 + \cos\pi - (0^2 + \cos(0))$

$= \pi^2 + -1 - 1 = \boxed{\pi^2 - 2}$

13. $\int_0^{\pi/2} (3\sin x - 2\cos x)\,dx$

$= -3\cos x - 2\sin x\Big]_0^{\pi/2} =$



$\left(-3\cos(\pi/2) - 2\sin(\pi/2)\right)$

$-\left(-3\cos(0) - 2\sin(0)\right) =$

$= -3(0) - 2(1) - (-3(1) - 2(0))$

$= -2 + 3 = \boxed{1}$

14. $\int_0^{\pi/4} \left(x - \sec^2 x\right) dx$

$= \frac{x^2}{2} - \tan x\Big]_0^{\pi/4} =$

$\frac{(\pi/4)^2}{2} - \tan(\pi/4) - (0 - \tan 0)$

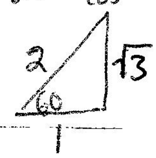
$\frac{\pi^2/16}{2} - 1 - 0 = \boxed{\frac{\pi^2}{32} - 1}$

15. $\int_0^{\pi/3} \sec\theta\tan\theta\,d\theta = \sec\theta\Big]_0^{\pi/3} =$

$\sec\pi/3 - \sec 0 =$

$2 - 1 = \boxed{1}$

$\sec = \frac{1}{\cos} = \frac{1}{A/H} = H/A$



$\sec = \frac{H}{A}$

$\cos = \frac{A}{H}$

16. Show that the Second Fundamental Theorem of Calculus holds for $F(x)=\int_1^x \frac{2}{t^2}\,dt$. Take the integral, then take the derivative.

$$\int_1^x \frac{2}{t^2}dt = \int_1^x 2t^{-2}dt = \frac{2t^{-1}}{-1}\Big]_1^x = \frac{-2}{t}\Big]_1^x = \frac{-2}{x}-\frac{-2}{1} = \frac{-2}{x}+2$$

$$\frac{d}{dx}\left(\frac{-2}{x}+2\right) = \frac{d}{dx}\left(-2x^{-1}+2\right) = 2x^{-2} = \boxed{\frac{2}{x^2}}$$

yes! $F'(x)=\frac{d}{dx}\int_1^x \frac{2}{t^2}dt = \boxed{\frac{2}{x^2}}$

17. Use the Second Fundamental Theorem of Calculus to find the derivatives of the following functions.

a) $f(x)=\int_1^x \left(t^2+1\right)^{20} dt$

$f'(x)=(x^2+1)^{20}$

b) $g(x)=\int_{-1}^x \sqrt{t^3+1}\,dt$

$g'(x)=\sqrt{x^3+1}$

c) $g(x)=\int_\pi^x \frac{1}{1+t^4}\,dt$

$g'(x)=\frac{1}{1+x^4}$

d) $f(x)=\int_4^{x^2} \cos\left(t^2\right) dt$

$f'(x)=\cos((x^2)^2)(2x) = 2x\cos(x^4)$

18. Find the interval on which the curve $y=\int_0^x \left(t^3+t^2+1\right) dt$ is concave up. Justify your answer.

y is concave up when
y' is increasing and
y'' is positive
y'' ← this is easiest one to see given an equation! Do sign chart w/ 2nd derivative of y.

$y'=x^3+x^2+1$
$y''=3x^2+2x$
$y''=3x^2+2x=0$
$x(3x+2)=0$
$x=0,\ x=-\frac{2}{3}$

	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	1
y''	$-(-)$ $+$		$-(+)$ $-$		$+(+)$ $+$

y is concave up on $(-\infty,-\frac{2}{3})\cup(0,\infty)$ since $y''>0$ on those intervals.

19. Your graphing calculator will evaluate $\int_a^b f(x)dx$ for you! $\boxed{\text{FnInt}}$ located as #9 in the $\boxed{\text{Math}}$ menu. The syntax is FnInt(function in x, x, lower, upper). For example. $\int_1^2 x^2dx$ would be FnInt(x^2, x, 1, 2) yielding 2.333. Your calculator is finding the definite integral by performing a Riemann Sum many, many times.

Check #1-15 using $\boxed{\text{FnInt}}$