Homework

Integrate using the given substitution.

1) $\int \frac{20x^4}{4x^5+3}\,dx;\ u = 4x^5+3$

$\frac{du}{dx} = 20x^4$

$dx = \frac{1}{20x^4}du$

$\int \frac{20x^4}{u}\,dx$

$\int \frac{1}{u}du$

$\ln|u| + C$

$\boxed{\ln|4x^5+3| + C}$

2) $\int 36x^2 e^{4x^3+3}\,dx;\ u = 4x^3+3$

$\frac{du}{dx} = 12x^2$

$dx = \frac{1}{12x^2}du$

$\int 36x^2 e^u\,dx$

$\int \frac{36\cancel{x^2}}{12\cancel{x^2}} e^u\,du$

$\int 3e^u du$

$3e^u + C$

$\boxed{3e^{4x^3+3} + C}$

Evaluate each indefinite integral.

5) $\int \frac{12x^2}{x^3+2}\,dx$

$u = x^3+2$

$\frac{du}{dx} = 3x^2$

$dx = \frac{1}{3x^2}du$

$\int \frac{12x^2}{u}\,dv$

$\int \frac{12\cancel{x^2}}{3\cancel{x^2}} \cdot \frac{1}{u}\,du$

$\int \frac{4}{u}du$

$4\ln|u| + C$

$\boxed{4\ln|x^3+2| + C}$

6) $\int \frac{20e^{5x}}{e^{5x}+3}\,dx$

$u = e^{5x}+3$

$\frac{du}{dx} = e^{5x}\cdot 5$

$dx = \frac{1}{5e^{5x}}du$

$\int \frac{20e^{5x}}{u}\,dx$

$\int \frac{20\cancel{e^{5x}}}{5\cdot u\cdot \cancel{e^{5x}}}\,du$

$\int \frac{4}{u}\,du$

$4\ln|u| + C$

$\boxed{4\ln|e^{5x}+3| + C}$

5. $\int \cos(2x+1)\,dx$

$u = 2x+1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$\int \cos(u)\,dx$

$\int \cos(u)\,\frac{1}{2}\cdot du$

$\int \frac{1}{2}\cos(u)\,du$

$\frac{1}{2}\sin(u) + C$

$\boxed{\frac{1}{2}\sin(2x+1) + C}$

6. $\int \sin^{10}x\cos x\,dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{1}{\cos x}$

$\int u^{10}\cos x\,dx$

$\int u^{10}\frac{\cancel{\cos x}}{\cancel{\cos x}}\,du$

$\int u^{10}du$

$\frac{1}{11}u^{11} + C$

$\boxed{\frac{1}{11}(\sin x)^{11} + C}$

7. $\displaystyle\int \frac{\sin x}{(\cos x)^5}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{1}{\sin x}$

$\int \frac{\sin x}{u^5}\,dx$

$\int \frac{\cancel{\sin x}}{u^5} \cdot -\frac{1}{\cancel{\sin x}}\,du$

$\int -\frac{1}{u^5}\,du$

$\int -u^{-5}\,du$

$\frac{1}{4}u^{-4} + C$

$\boxed{\frac{1}{4}(\cos x)^{-4} + C}$

8. $\displaystyle\int \frac{(\sqrt{x}-1)^2}{\sqrt{x}}\,dx$

$u = \sqrt{x} - 1$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}\ \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{u^2}{\sqrt{x}}\,dx$

$\int \frac{u^2}{\cancel{\sqrt{x}}} \cdot \frac{2\cancel{\sqrt{x}}}{1} \cdot du$

$\int 2u^2 \cdot du$

$\frac{2}{3}u^3 + C$

$\boxed{\frac{2}{3}(\sqrt{x}-1)^3 + C}$

9. $\displaystyle\int \sqrt{x^3+x^2}(3x^2+2x)\,dx$

$u = x^3 + x^2$

$\frac{du}{dx} = 3x^2 + 2x$

$dx = \frac{1}{3x^2+2x}\,du$

$\int \sqrt{u}\,(3x^2+2x)\,dx$

$\int \sqrt{u}\left(\frac{3x^2+\cancel{2x}}{\cancel{3x^2}+2x}\right) \cdot du$

$u^{\frac{1}{2}}$ $\quad \int \sqrt{u}\,du$

$\frac{2}{3}u^{\frac{3}{2}} + C$

$\boxed{\frac{2}{3}(x^3+x^2)^{\frac{3}{2}} + C}$