Determine $\int_0^7 f(x)dx\ if\ f(x) = \begin{cases} x+5, & x<2 \\ 2x, & x>2 \end{cases}$

$\int_0^2 x+5dx = \frac{1}{2}x^2+5x \Big|_0^2\ \left[\frac{1}{2}(2)^2+5(2)\right] - \cancel{\left[\frac{1}{2}(0)^2+5(0)\right]}$ $2+10=12$

$\int_2^7 2xdx = x^2 \Big|_2^7\ 7^2-2^2 = 49-4 = 45$

$12+45 = 57$

★ Plug in 0 and 3 to see if it goes negative!

6. A particle moves along the x-axis. The velocity of the particle at time t is $6t-t^2$. What is the total distance traveled by the particle from time $t=0$ to $t=3$?

(A) 3 (B) 6 (C) 9 [✓ (D) 18] (E) 27

$\int_0^3 6t-t^2\,dt = 3t^2-\frac{1}{3}t^3 \Big|_0^3\ \overset{27}{3(3)^2}-\overset{9}{\frac{1}{3}(3)^3}$

$=18$

f(x) increasing

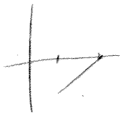
86. If $f'(x)>0$ for all real numbers x and $\int_4^7 f(t)dt=0$, which of the following could be a table of values for the function f?

x goes fr. 4-7

f(x) has displacement 0

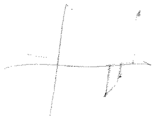
~~(A)~~

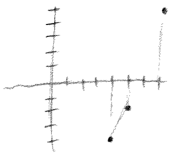
x	$f(x)$
4	−4
5	−3
7	0



[✓ (B)]

x	$f(x)$
4	−4
5	−2
7	5





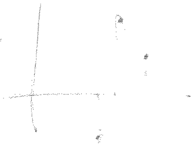
(some neg + pos f(x) values

→ balance

~~(C)~~

x	$f(x)$
4	−4
5	6
7	3

not inc



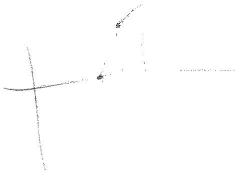
~~(D)~~

x	$f(x)$
4	0
5	0
7	0

not inc

~~(E)~~

x	$f(x)$
4	0
5	4
7	6



$a(t)$ + $v(t)$ inc

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 [✓ (E) 11.710]

(0, 5)
(3, ?)

$$5 + \int_0^3 \frac{t+2}{\sqrt{t^3+1}}\,dt$$

(6.71

$t = 2x \quad \frac{dt}{dx} = 2 \quad \frac{1}{2}dt = dx$

90. Let f be a function such that $\int_6^{12} f(2x)\,dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t)\,dt = 5$

$\frac{1}{2}\int_{12}^{24} f(t)\,dt = 10$

[✓ (B) $\int_{12}^{24} f(t)\,dt = 20$]

$\int_{12}^{24} f(t)\,dt = 20$

(C) $\int_6^{12} f(t)\,dt = 5$

(D) $\int_6^{12} f(t)\,dt = 20$

(E) $\int_3^6 f(t)\,dt = 5$

$\frac{du}{dx} = 2 \quad \frac{1}{2}du = dx$

11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ [✓ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$] (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

$\frac{1}{2}\int_1^5 \sqrt{u}\,du$

Homework

Below is a table of f(x), g(x), and their derivatives at certain x values. It is known that f(x), g(x), and their derivatives are continuous and differentiable for all x values.

x	-5	-2	3	8
f(x)	2	-5	-2	3
f'(x)	-2	3	1	5
g(x)	3	-5	-2	9
g'(x)	7	2	1	0

$\int_{-5}^{-2} f'(x)dx$

$f(-2) - f(-5) =$

$-5 - 2 = \boxed{-7}$

$\int_{3}^{8} g'(x)dx$

$g(8) - g(3) =$

$9 + +2 = \boxed{11}$

$\int_{8}^{-5} g'(x)dx$

$g(-5) - g(8) =$

$3 - 9 = \boxed{-6}$

$\int_{-2}^{3} f'(x) + g'(x)dx$

$[f(3) - f(-2)] + [g(3) - g(-2)]$

$-2 + +5 \quad + \quad -2 + +5$

$3 + 3 = \boxed{6}$

$\int_{-5}^{3} f'(g(x))g'(x)dx$

$f(g(x)) \Big|_{-5}^{3} \quad f(\overset{-2}{g(3)}) - f(\overset{3}{g(-5)})$

$-5 - +2 = \boxed{-3}$

$\int_{3}^{8} g'(f(x))f'(x)dx$

$g(f(x)) \Big|_{3}^{8} \quad g(\overset{3}{f(8)}) - g(\overset{-2}{f(3)})$

$-2 - -5 = \boxed{3}$

$\frac{d}{dx}\frac{f(x)}{g(x)}$

$$\boxed{\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$$

$\frac{d}{dx}2f(g(x)) + x^2$

$$\boxed{2f'(g(x))g'(x) + 2x}$$

$u = -4x$ $\frac{du}{dx} = -4$

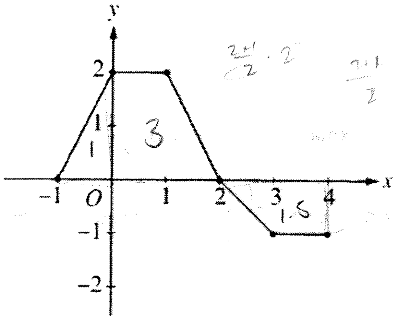
$\int_0^1 e^{-4x}\,dx =$ $-\frac{1}{4}\int_0^{-4} e^u\,du = -\frac{1}{4}e^u\Big|_0^{-4} -\frac{1}{4}\left[e^{-4} - e^0\right]$

$e^{-4} - 1$

$-\frac{1}{4}e^{-4} + \frac{1}{4}$

$-\frac{e^{-4}-1}{4}$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ [✓ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$] (E) $4 - 4e^{-4}$



$\frac{2+1}{2} \cdot 2$

$\frac{2+1}{2}$ $\frac{3}{2} \cdot 1$

$4 - 1.5 =$

The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$? $F(4) - F(-1)$

(A) 1 [✓ (B) 2.5] (C) 4 (D) 5.5 (E) 8

$12 - 4 = 8$

If $\int_1^5 F(x)\,dx = 12$ and $\int_1^3 F(x)\,dx = 4$, then $\int_3^5 F(x)\,dx =$

A) - 12 B) - 8 [✓ C) 8] D) 12 E) 20

Use a right hand Riemann sum with 3 subintervals of equal width to approximate the value of $\int_0^6 f(x)\,dx$. Selected values of the function are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	1	2	5	10	17	26	37

~~$1(37+26+17+10+5+2)$~~

~~$\begin{array}{r}17\\ \times 2\\ \hline 34\end{array}$~~

~~$34 + 26$~~

$\begin{array}{r}60\\ +37\\ \hline 97\end{array}$

[✓ A) 118] B) 76 C) 46 D) 38 E) 18

$\begin{array}{r}17\\ +5\\ \hline 22\end{array}$

$2(37 + 17 + 5)$

$\begin{array}{r}22\\ +37\\ \hline 59\end{array}$ $\begin{array}{r}59\\ \times 2\\ \hline 118\end{array}$

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Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

7 23 12 7

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

$(6, 62)$ $(8, 55)$ $\frac{62-55}{6-8} = \frac{-7}{-2} = \boxed{\frac{7}{2} \text{ °C/cm}}$

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

$T''(x) > 0$

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

↳ CCU

c) $\int_0^8 T'(x)dx = T(x)\Big|_0^8 = T(8) - T(0)$

$55 - 100 = -45°C$

From 0 to 8 minutes, the total change in the temperature of the wire is -45°C.

↳ decreases by 45°C

∪

d) No because $T'(x) = 7$ at two points in the interval, so $T''(x)$ has to equal zero at one point. This means it not always positive.

$$\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e x+\frac{1}{x}\,dx$$

$$= \left(\frac{1}{2}x^2+\ln|x|\right)\Big|_1^e$$

$$= \left(\frac{e^2}{2}+\ln(e)\right)-\left(\frac{1}{2}+\ln(1)\right)$$

$$= \left(\frac{e^2}{2}+1\right)-\frac{1}{2}$$

$$= \frac{e^2}{2}+\frac{1}{2}$$

$$= \frac{e^2+1}{2}$$

A $\frac{e^2-1}{2}$

[✓ B] $\frac{e^2+1}{2}$

C $\frac{e^2+2}{2}$

D $\frac{e^2-1}{e^2}$

E $\frac{2e^2-8e+6}{3e}$

An antiderivative for $\frac{1}{x^2-2x+2}$ is

A $-\left(x^2-2x+2\right)^{-2}$

B $\ln\left(x^2-2x+2\right)$

C $\ln\left|\frac{x-2}{x+1}\right|$

$$\int \frac{1}{x^2-2x+2}\,dx$$

complete the square

$$= \int \frac{1}{(x-1)^2+1}\,dx$$

D arcsec(x-1) $= \tan^{-1}(x-1)+C$

[✓ E] arctan(x-1)

Challenging Problem

$$\int \frac{x+4}{x^2+2x+5}\,dx = \int \frac{x+4}{(x+1)^2+4}\,dx$$

$u = x+1$

$du = dx$

$$= \int \frac{u+3}{u^2+4}\,du = \int \frac{u}{u^2+4}\,du + \int \frac{3}{u^2+4}\,du$$

$$= \frac{1}{2}\ln\left(u^2+4\right)+\frac{3}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$$

$$= \frac{1}{2}\ln\left((x+1)^2+4\right)+\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$$

Video + Notes