HW 6.9

P. 354 #1-9 odd, 17-25 odd, 47

1. $\int x\sin(x)dx$ $\qquad u = x \qquad dv = \sin(x)dx$

$\qquad du = dx \qquad v = -\cos(x)$

$$\int x\sin(x)dx = -x\cos(x) - \int -\cos(x)dx = -x\cos(x) + \int \cos(x)dx = -x\cos(x) + \sin(x) + C$$

3. $\int 3te^{2t}dt$ $\qquad u = 3t \qquad dv = e^{2t}dt$

$\qquad du = 3dt \qquad v = \frac{1}{2}e^{2t}$

← don't forget

$$\int 3te^{2t}dt = 3t\cdot\frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t}\cdot 3dt$$

$$= \frac{3t}{2}e^{2t} - \frac{3}{2}\int e^{2t}dt = \frac{3t^{2t}}{2}e^{2t} - \frac{3}{2}\cdot\frac{1}{2}e^{2t} + C = \frac{3t^{2t}}{2}e^{2t} - \frac{3}{4}e^{2t} + C$$

5. $\int x^2\cos(x)dx$ $\qquad u = x^2 \qquad dv = \cos(x)dx$

$\qquad du = 2xdx \qquad v = \sin(x)$

$$\int x^2\cos(x)dx = x^2\sin(x) - \int \sin(x)2x\,dx \qquad u = 2x \qquad dv = \sin(x)dx$$

$$= x^2\sin(x) - \left(-2x\cos(x) - \int -\cos(x)2dx\right) \qquad du = 2dx \qquad v = -\cos(x)dx$$

$$= x^2\sin(x) + 2x\cos(x) - 2\int\cos(x)dx$$

$$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$$

OR Tabular Method

$\frac{d}{dx}f(x)$		$\int g(x)$
x^2	+	$\cos(x)$
$2x$	−	$\sin(x)$
2	+	$-\cos(x)$
0		$-\sin(x)$

7. $\int y\ln y\,dy$ $\qquad u = \ln y \qquad dv = y\,dy$

$\qquad du = \frac{1}{y}dy \qquad v = \frac{1}{2}y^2$

$$\int y\ln y\,dy = \ln(y)\frac{1}{2}y^2 - \int \frac{1}{2}y^2\cdot\frac{1}{y}dy = \frac{1}{2}\ln(y)y^2 - \frac{1}{2}\int y\,dy = \frac{1}{2}\ln(y)y^2 - \frac{1}{4}y^2 + C$$

9. $\int \log_2(x)dx$ $\qquad u = \log_2(x) \qquad dv = dx$

$\qquad du = \frac{1}{\ln(2)x}dx \qquad v = x$

$$\int \log_2(x)dx = \log_2(x)x - \int x\cdot\frac{1}{\ln(2)x}dx = \log_2(x)x - \int\frac{1}{\ln(2)}dx = \log_2(x)x - \frac{1}{\ln(2)}x + C$$

Equivalent to book answer

17. $\int e^x \sin(x)\,dx$ $\qquad u=\sin(x) \qquad dv=e^x\,dx$

$\int e^x \sin(x)\,dx = \sin(x)e^x - \int e^x\cos(x)\,dx$ $\qquad du=\cos(x)\,dx \qquad v=e^x$

$u=\cos(x) \qquad dv=e^x\,dx$

$\int e^x\sin(x)\,dx = e^x\sin(x) - \left(e^x\cos(x) - \int -e^x\sin(x)\,dx\right)$ $\qquad du=-\sin(x)\,dx \qquad v=e^x$

$\int e^x\sin(x)\,dx = e^x\sin(x) - e^x\cos(x) - \int e^x\sin(x)\,dx$

$2\int e^x\sin(x)\,dx = e^x\sin(x) - e^x\cos(x)$

$\int e^x\sin(x)\,dx = \dfrac{e^x\sin(x) - e^x\cos(x)}{2} + C$

19. $\int e^x\cos(2x)\,dx$ $\qquad u=\cos(2x) \qquad dv=e^x\,dx$

$\int e^x\cos(2x)\,dx = e^x\cos(2x) - \int e^x(-2\sin(2x))\,dx$ $\qquad du=-2\sin(2x)\,dx \qquad v=e^x$

$\int e^x\cos(2x)\,dx = e^x\cos(2x) + 2\int e^x\sin(2x)\,dx$ $\qquad u=\sin(2x) \qquad dv=e^x\,dx$

$\int e^x\cos(2x)\,dx = e^x\cos(2x) + 2\left(e^x\sin(2x) - \int e^x\, 2\cos(2x)\,dx\right)$ $\qquad du=2\cos(2x) \qquad v=e^x$

$\int e^x\cos(2x)\,dx = e^x\cos(2x) + 2e^x\sin(2x) - 4\int e^x\cos(2x)\,dx$

$5\int e^x\cos(2x)\,dx = e^x\cos(2x) + 2e^x\sin(2x)$

$\int e^x\cos(2x)\,dx = \dfrac{e^x\cos(2x) + 2e^x\sin(2x)}{5} + C$

21. $\int x^4 e^{-x}\,dx = -x^4e^{-x} - 4x^3e^{-x} - 12x^2e^{-x} - 24xe^{-x} - 24e^{-x} + C$

$\frac{d}{dx} f(x)$		$\int g(x)$
x^4	+	e^{-x}
$4x^3$	−	$-e^{-x}$
$12x^2$	+	e^{-x}
$24x$	−	$-e^{-x}$
24	+	e^{-x}
0		$-e^{-x}$

23. $\int x^3 e^{-2x}\,dx$

$\frac{d}{dx} f(x)$		$\int g(x)$
x^3	+	e^{-2x}
$3x^2$	−	$-\frac{1}{2}e^{-2x}$
$6x$	+	$\frac{1}{4}e^{-2x}$
6	−	$-\frac{1}{8}e^{-2x}$
0		$\frac{1}{16}e^{-2x}$

$\int x^3e^{-2x}\,dx = -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{6}{8}xe^{-2x} - \frac{6}{16}e^{-2x} + C$

25. $\int_0^{\pi/2} x^2 \sin(2x) =$

$\frac{d}{dx} f(x)$		$\int g(x)$
x^2	$+$	$\sin(2x)$
$2x$	$-$	$-\frac{1}{2}\cos(2x)$
2	$+$	$-\frac{1}{4}\sin(2x)$
0		$\frac{1}{8}\cos(2x)$

$$\int_0^{\pi/2} x^2 \sin(2x)dx = \left[-\frac{x^2}{2}\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)\right]\Big|_0^{\pi/2}$$

$$= \left[\frac{-\pi^2}{8}\cos(\pi) + \frac{\pi}{4}\sin(\pi) + \frac{1}{4}\cos(\pi)\right] - \left[0 + 0 + \frac{1}{4}\cos(0)\right]$$

$$= \frac{\pi^2}{8} + 0 - \frac{1}{4} - \frac{1}{4} = \frac{\pi^2}{8} - \frac{1}{2}$$

47. $\int x^n \cos(x)dx =$

$$\left\{ \begin{array}{ll} u = x^n & dv = \cos(x)dx \\ du = nx^{n-1}\,dx & v = \sin(x) \end{array} \right.$$

$$= x^n \sin(x) - \int \sin(x) n x^{n-1} dx$$

$$= x^n \sin(x) - n\int x^{n-1} \sin(x)dx \quad \checkmark$$