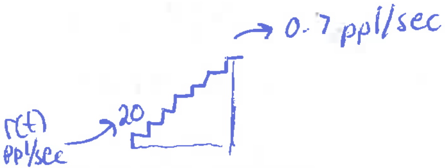
Calc Ok

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$



where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

a) $\int_0^{300} r(t)dt = 270$ ppl

b) This is total amount!

$$P(t) = \underset{\substack{\uparrow \\ \text{initial}}}{20} + \underset{\substack{\uparrow \\ \text{in}}}{\int_0^{300} r(t)dt} - \underset{\substack{\uparrow \\ \text{out}}}{\int_0^{300} 0.7\,dt} = 80 \text{ ppl}$$

c) After $t = 300$, no one joins the line...

$80 - \int_{300}^{t} 0.7dx = 0$

$80 - (0.7x)\Big|_{300}^{t} = 0 \rightarrow 80 - [0.7t - 210] = 0$

$290 - 0.7t = 0$

$t = 414.285$ seconds

OR $\frac{80 \text{ ppl}}{0.7 \text{ ppl/sec}} = 114.286$ sec

time to clear the last 80 ppl ↓

$300 + 114.286 = \boxed{414.286}$ ↙ seconds

d) EPs

$t = 0 \quad P(0) = 20$

$t = 300 \quad P(300) = 80$

CP:

$P'(t) = r(t) - 0.7 = 0$

$r(t) = 0.7$ graph! Find intersections

$t_1 = 33.013298 \quad t_2 = 166.57472$

$P(t_1) = 3.803$

$P(t_2) = 158.070$

The number of people in line is at a min when $t = 33.013$ sec when 4 people are in line.

Non-Calc

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

Position of P

→ initial position of Q

↳ velocity of Q

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

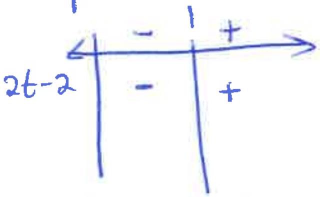
(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

a) P is moving left when V_p is negative

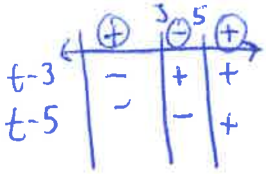
V_p is derivative of $X_p(t)$: $V_p(t) = \frac{1}{t^2-2t+10}(2t-2) = \frac{2t-2}{t^2-2t+10}$

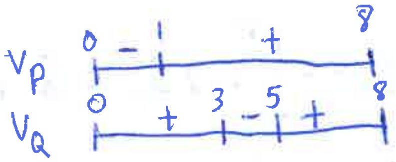
$t^2-2t+10$ is always +



$V_p(t)$ is − on $[0,1)$. Particle moves left on $[0,1)$

b) $V_Q = (t-3)(t-5)$





Particles are traveling the same direction on $(1,3)$ and $(5,8]$ since V_p and V_Q have the same sign.

c) $V_Q'(t) = 2t-8$ $\quad V_Q(2) = 3$

$a_Q(2) = -4$

At $t=2$, the speed of Particle Q is decreasing since $a_Q(2)$ and $V_Q(2)$ have different signs.

d) Q changes direction at $t=3$ (from sign chart above).

$P_Q(3) = 5 + \int_0^3 t^2-8t+15\,dt$

$P_Q(3) = 5 + \left(\frac{1}{3}t^3 - 4t^2 + 15t\right)\Big|_0^3 = 5 + (9-36+45) = 23$

AP CollegeBoard AP Calculus BC Test Booklet

Unit 8 Review MC (no areas and volumes)

Name

Calc **1.** The velocity, in ft/sec, of a particle moving along the *x*-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

$$\frac{1}{3}\int_0^3 v(t)dt = 20.086$$

● A 20.086 ft/sec

○ B 26.447 ft/sec

○ C 32.809 ft/sec

○ D 40.671 ft/sec

○ E 79.342 ft/sec

Calc **2.** What is the average value of $y = \frac{\cos x}{x^2+x+2}$ on the closed interval $[-1,\ 3]$?

$$\frac{1}{3-(-1)}\int_{-1}^{3}\frac{\cos(x)}{(x^2+x+2)}dx = 0.183$$

○ A −0.085

○ B 0.090

● C 0.183

○ D 0.244

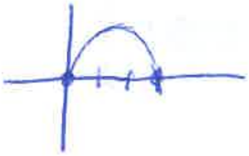
○ E 0.732

3. What is the average value of y for the part of the curve $y = 3x - x^2$, which is the *first quadrant*?

$y = x(3-x)$

$0 = x(3-x)$

$x = 0, 3$



$$A_{vg} = \frac{1}{3-0}\int_0^3 3x - x^2 dx$$

$$= \frac{1}{3}\left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]\Big|_0^3 = \frac{1}{3}\left[\frac{27}{2} - 9\right] = \frac{9}{2} - 3 = \boxed{\frac{3}{2}}$$

(A) −6

(B) −2

[✓ (C)] $\frac{3}{2}$

(D) $\frac{9}{4}$

(E) $\frac{9}{2}$

Calc

4. If $f(x) = (x + 2)\sin\left(\sqrt{x+2}\right)$, what is the average value of f on the closed interval [0, 6]?

$$\frac{1}{6}\int_0^6 (x+2)\sin\left(\sqrt{x+2}\right)\,dx = 3.348$$

(A) 2.220

[✓ (B)] 3.348

(C) 4.757

(D) 20.090

(E) 28.541

5.

Time (weeks)	0	2	6	10
Level	210	200	190	180

The table above gives the level of a person's cholesterol at different times during a 10-week treatment period. What is the average level over this 10-week period obtained by using a trapezoidal approximation with the subintervals [0, 2], [2, 6], and [6, 10] ?

approximates the integral

$$\frac{(y_1+y_2)\Delta x}{2}$$

$$\frac{1}{10}\int_0^{10} \text{Level}\,dt \simeq \frac{1}{10}\left[\frac{(210+200)2}{2} + \frac{(200+190)4}{2} + \frac{(190+180)4}{2}\right]$$

$$\simeq \frac{1}{10}\left[410 + (390)2 + (370)2\right]$$

$$\simeq \frac{1}{10}\left[410 + 780 + 740\right] = \frac{1}{10}\left[1930\right] = 193$$

$$\begin{array}{r} {}^{1} \\ 410 \\ 780 \\ +740 \\ \hline 1930 \end{array}$$

- (A) 188
- [✓ (B) 193]
- (C) 195
- (D) 198

Calc **6.** A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

- (A) $\frac{a(8)-a(0)}{8}$ → average jerk
- [✓ (B) $\frac{1}{8}\int_0^8 v(t)\,dt$]
- (C) $\frac{1}{8}\int_0^8 |v(t)|\,dt$ → average speed
- (D) $\frac{1}{2}\int_0^8 v(t)\,dt$ → half displacement
- (E) $\frac{v(0)+v(8)}{2}$ → Nothing

If given position: $\frac{P(8)-P(0)}{8-0}$

but not given position!

So: $\frac{1}{8}\int_0^8 v(t)dt$

Calc **7.** A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos(2 - t^2)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

initial position

$v(t)=0$

$\cos(2-t^2)=0$

graph it!

$t=0.6551364$

$P(t)=P(0)+\int_0^t v(x)dx$

$P(0.6551364)=3+\int_0^{0.6551364}\cos(2-t^2)dt=2.816$

Ⓐ 0.411

Ⓑ 1.310

● 2.816

Ⓓ 3.091

Ⓔ 3.411

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8. A particle moves along the *x*-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

initial position

Ⓐ 4

● 6

Ⓒ 9

Ⓓ 11

Ⓔ 12

$$P(1) = P(0) + \int_0^1 v(t)\,dt$$

$$P(1) = 2 + \int_0^1 3t^2 + 6t\,dt$$

$$P(1) = 2 + (t^3 + 3t^2)\Big|_0^1 = 2 + (1+3) = 6$$

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9. A particle travels along the *x*-axis so that at time $t \geq 0$ its velocity is given by $v(t) = t^2 - 6t + 8$. What is the total distance the particle travels from $t = 0$ to $t = 3$?

$$\int_0^3 |v(t)|\,dt = \int_0^2 t^2 - 6t + 8\,dt + \left|\int_2^3 t^2 - 6t + 8\,dt\right| = \left(\tfrac{1}{3}t^3 - 3t^2 + 8t\right)\Big|_0^2 + \left|\left(\tfrac{1}{3}t^3 - 3t^2 + 8t\right)\Big|_2^3\right|$$

$$= \left(\tfrac{8}{3} - 12 + 16\right) + \left|(9 - 27 + 24) - \left(\tfrac{8}{3} - 12 + 16\right)\right|$$

$$= \left(\tfrac{8}{3} + 4\right) + \left|6 - \left(\tfrac{8}{3} + 4\right)\right| = \left(\tfrac{20}{3}\right) + \left|6 - \tfrac{20}{3}\right|$$

$$= \frac{20}{3} + \frac{2}{3} = \boxed{\frac{22}{3}}$$

$v(t) = t^2 - 6t + 8$ does $v(t)$ become negative on $(0,3)$?

$v(t) = (t-4)(t-2)$

$t = 2, 4$

	2		4	
	⊕		⊖	⊕
$t-2$	−		+	+
$t-4$	−		−	+

$v(t)$ is + $[0,2)$

and − $(2,3]$

Ⓐ 1

Ⓑ 6

Ⓒ 20/3

● 22/3

Ⓔ 8

Calc

10. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2 \sin t$ for $0 \leq t \leq 2$. What is the total distance traveled by the particle between t = 0 and t = 2?

$\int_0^2 |v(t)| dt = 2.539$

Ⓐ -3.637

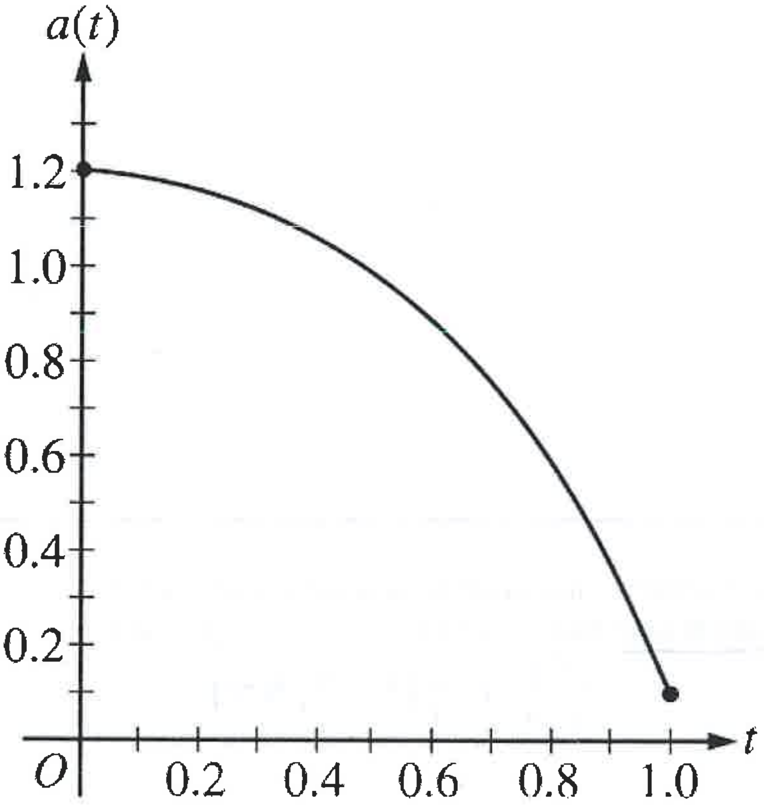
Ⓑ -1.973

Ⓒ 1.531

● 2.539

Ⓔ 3.637

Calc **11.**



A particle moves along the x-axis so that its acceleration $a(t)$ is given by the graph above for all values of t where $0 \le t \le 1$. At time $t = 0$, the velocity of the particle is $-\frac{1}{2}$. Which of the following statements must be true?

initial velocity

$v(t) = -\frac{1}{2} + \int_0^t a(x)dx$

from 0-1, $\int_0^1 a(x)dx$ is greater than $\frac{1}{2}$ so velocity ends up positive

~~(A)~~ The particle passes through $x = 0$ for some t between $t = 0$ and $t = 1$.

↳ no position info

[✓ (B)] The velocity of the particle is 0 for some t between $t = 0$ and $t = 1$.

~~(C)~~ The velocity of the particle is negative for all values of t between $t = 0$ and $t = 1$.

↳ not true

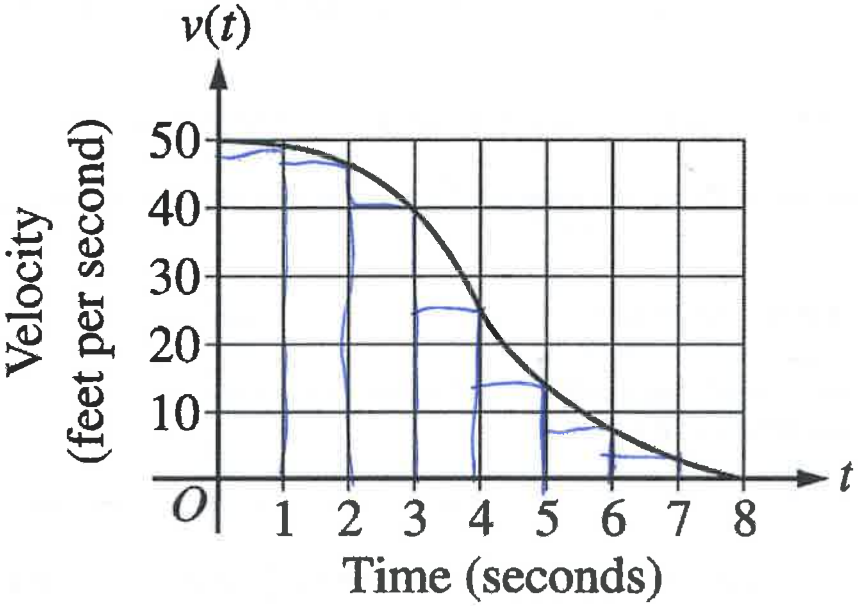
~~(D)~~ The velocity of the particle is positive for all values of t between $t = 0$ and $t = 1$.

↳ not true, starts negative

~~(E)~~ The velocity of the particle is less than $-\frac{1}{2}$ for all values of t between $t = 0$ and $t = 1$.

↳ not true, $a(t)$ is positive, $v(t)$ increases from $-\frac{1}{2}$

Calc **12.**



The graph above gives the velocity, in in ft/sec, of a car for $0 \leq t \leq 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

~~(A)~~ 21 ft

~~(B)~~ 26 ft

~~(C)~~ 180 ft

[✓ (D)] 210 ft

(E) 260 ft

$\int_0^8 |v(t)|dt$ but v(t) is always positive, anyways

use rectangles.

Right RAM: 49+46+40+25+14+8+3 = 185

but that's an underestimate!

13. A function f(t) gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)\,dt$ in the context described?

t=4 → 4pm

t=10 → 10pm

$\int f(t)dt$ → amount evaporated

Ⓐ The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon

● The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

Ⓒ The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

Ⓓ The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

Ⓔ The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

Calc **14.** A home uses fuel oil at the rate $r(t) = 10 + 8\,sin\left(\frac{t}{60}\right)$ gallons per day, where t is the number of days from the beginning of the heating season. To the nearest gallon, what is the total amount of fuel oil used from $t = 0$ to $t = 60$ days?

$$\int_0^{60} r(t)dt = 820.654 \text{ gal}$$

Ⓐ 7 gal

Ⓑ 14 gal

Ⓒ 600 gal

● 821 gal

Ⓔ 1004 gal

Calc **15.** The number of bacteria in a container increases at the rate of $R(t)$ bacteria per hour. If there are
initial → 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

$$B(3) = \underset{\substack{\uparrow \\ \text{initial}}}{1000} + \int_0^3 R(t)dt$$

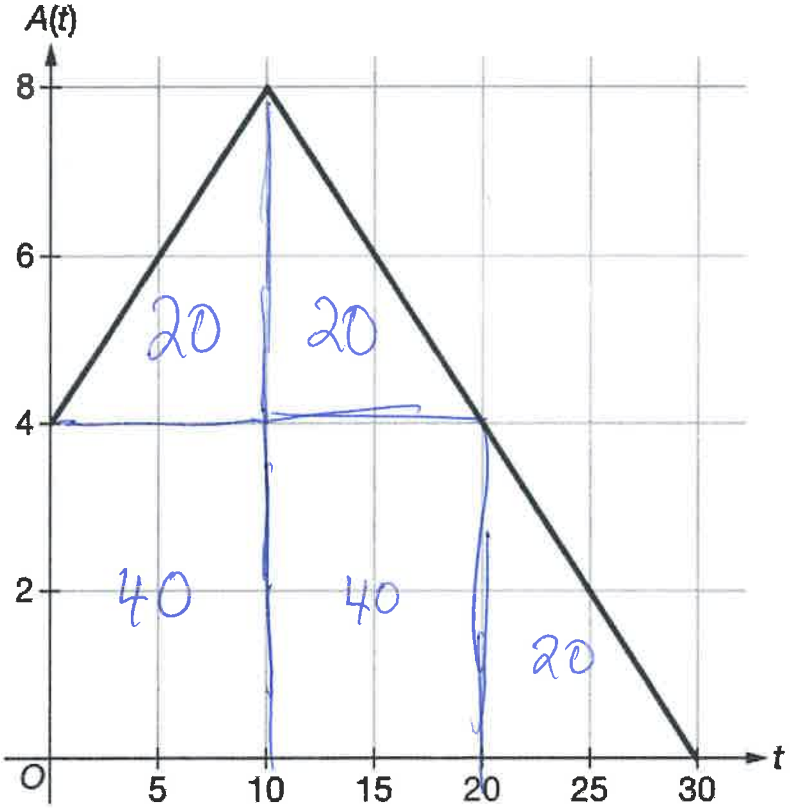
Ⓐ $R(3)$

Ⓑ $1000 + R(3)$

Ⓒ $\int_0^3 R(t)\,dt$

● $1000+ \int_0^3 R(t)\,dt$

16.



The rate at which ants arrive at a picnic is modeled by the function A, where $A(t)$ is measured in ants per minute and t is measured in minutes. The graph of A for $0 \le t \le 30$ is shown in the figure above. How many ants arrive at the picnic during the time interval $0 \le t \le 30$?

$\int_0^{30} A(t)dt = 140$ ants

Ⓐ 8

Ⓑ 70

Ⓒ 120

● 140