

## HW 8.9

Pg. 424 #1-13, 23

1.  $y = x^2 \quad \frac{dy}{dx} = 2x \quad L = \int_{-1}^2 \sqrt{1 + (2x)^2} dx = 6.126$

2.  $y = \tan(x) \quad \frac{dy}{dx} = \sec^2(x) \quad L = \int_{-\pi/3}^0 \sqrt{1 + (\sec^2(x))^2} dx = 2.057$

3.  $x = \sin(y) \quad \frac{dx}{dy} = \cos(y) \quad L = \int_0^{\pi} \sqrt{1 + \cos^2(y)} dy = 3.820$

4.  $x = \sqrt{1-y^2} \quad \frac{dx}{dy} = \frac{1}{2}(1-y^2)^{-1/2}(-2y) = -y(1-y^2)^{-1/2}$   
 $L = \int_{-1/2}^{1/2} \sqrt{1 + [-y(1-y^2)^{-1/2}]^2} dy = 1.047$

5.  $y^2 + 2y = 2x + 1$   
 $y^2 + 2y - 1 = 2x$   
 $\frac{1}{2}y^2 + y - \frac{1}{2} = x \quad \frac{dx}{dy} = y + 1$   
 $L = \int_{-1}^3 \sqrt{1 + (y+1)^2} dy = 9.294$

6.  $y = \sin(x) - x \cos(x)$   
 $\frac{dy}{dx} = \cos(x) - x(-\sin(x)) - \cos(x) = x \sin(x)$   
 $L = \int_0^{\pi} \sqrt{1 + (x \sin(x))^2} dx = 4.698$

7.  $y = \int_0^x \tan(t) dt \quad L = \int_0^{\pi/6} \sqrt{1 + \tan^2(x)} dx = 0.549$   
 $\frac{dy}{dx} = \tan(x)$

8.  $x = \int_0^y \sqrt{\sec^2(t) - 1} dt \quad L = \int_{-\pi/3}^{\pi/4} \sqrt{1 + (\sec^2(y) - 1)} dy = 2.198$   
 $\frac{dx}{dy} = \sqrt{\sec^2(y) - 1}$

9.  $y = \sec(x) \quad L = \int_{-\pi/3}^{\pi/3} \sqrt{1 + \sec^2(x) \tan^2(x)} dx = 3.139$   
 $\frac{dy}{dx} = \sec(x) \tan(x)$

10.  $y = \frac{(e^x + e^{-x})}{2} \quad \frac{dy}{dx} = \frac{e^x - e^{-x}}{2}$   
 $L = \int_{-3}^3 \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2} dx = 20.036$

1. 5. 101

$\sqrt{2} = \sqrt{1+1} = \sqrt{1^2+1^2}$

$$\sqrt{1^2+1^2} = \sqrt{1^2+1^2} \Rightarrow \sqrt{1^2+1^2} = \sqrt{1^2+1^2}$$

1

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2

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3

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4

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5

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6

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7

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8

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9

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10

$$11. \quad y = \frac{1}{3}(x^2+2)^{3/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+2)^{1/2} \cdot 2x = (x^2+2)^{1/2} x$$

$$\left(\frac{dy}{dx}\right)^2 = (x^2+2)x^2 = x^4+2x^2$$

$$L = \int_0^3 \sqrt{1+x^4+2x^2} dx = \int_0^3 \sqrt{x^4+2x^2+1} dx = \int_0^3 \sqrt{(x^2+1)^2} dx = \int_0^3 x^2+1 dx$$

$$= \left(\frac{1}{3}x^3+x\right) \Big|_0^3 = 12$$

$$12. \quad y = x^{3/2} \quad \frac{dy}{dx} = \frac{3}{2}x^{1/2} \quad \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}x$$

$$L = \int_0^4 \sqrt{1+\frac{9}{4}x} dx = \int_0^4 \left(1+\frac{9}{4}x\right)^{1/2} dx = \left(\frac{2}{3}\left(1+\frac{9}{4}x\right)^{3/2} \cdot \frac{4}{9}\right) \Big|_0^4 = \left(\frac{8}{27}\left(1+\frac{9}{4}x\right)^{3/2}\right) \Big|_0^4$$

$$= \frac{8}{27}(10)^{3/2} - \frac{8}{27}(1)^{3/2} = \frac{8}{27}(10\sqrt{10}) - \frac{8}{27}$$

$$13. \quad x = \frac{y^3}{3} + \frac{1}{4y} \quad \frac{dx}{dy} = y^2 - \frac{1}{4y^2} \quad \left(\frac{dx}{dy}\right)^2 = \left(y^4 - \frac{1}{4} - \frac{1}{4} + \frac{1}{16y^4}\right) = y^4 - \frac{1}{2} + \frac{1}{16y^4}$$

$$L = \int_1^3 \sqrt{1+y^4 - \frac{1}{2} + \frac{1}{16y^4}} dy = \int_1^3 \sqrt{y^4 + \frac{1}{16y^4} + \frac{1}{2}} dy = \int_1^3 \frac{1}{4y^2} \sqrt{16y^8 + 1 + 8y^4} dy$$

$$= \int_1^3 \frac{1}{4y^2} \sqrt{16y^8 + 8y^4 + 1} dy = \int_1^3 \frac{1}{4y^2} \sqrt{(4y^4+1)^2} dy = \int_1^3 \frac{1}{4y^2} (4y^4+1) dy$$

$$= \int_1^3 y^2 + \frac{1}{4y^2} dy = \int_1^3 y^2 + \frac{1}{4}y^{-2} dy = \left(\frac{1}{3}y^3 - \frac{1}{4}y^{-1}\right) \Big|_1^3 = \left(\frac{1}{3}y^3 - \frac{1}{4y}\right) \Big|_1^3$$

$$= \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \left(\frac{108}{12} - \frac{1}{12}\right) - \left(\frac{4}{12} - \frac{3}{12}\right)$$

$$= \frac{107}{12} - \frac{1}{12} = \frac{106}{12} = \frac{53}{6}$$

23. "How wide" becomes length along the curve.

$$L = \int_0^{20} \sqrt{1 + \left[\frac{3\pi}{20} \cos\left(\frac{3\pi}{20}x\right)\right]^2} dx$$

$$y = \sin\left(\frac{3\pi}{20}x\right)$$

$$\frac{dy}{dx} = \cos\left(\frac{3\pi}{20}x\right) \cdot \frac{3\pi}{20}$$

$$L = 21.068 \text{ in} \approx \boxed{21.07 \text{ in}}$$

11/5/20

1.  $\sin^{-1}(\sin \frac{5\pi}{6}) = \frac{5\pi}{6}$

2.  $\cos^{-1}(\cos \frac{7\pi}{6}) = \frac{7\pi}{6}$

3.  $\tan^{-1}(\tan \frac{3\pi}{4}) = \frac{3\pi}{4}$

4.  $\cot^{-1}(\cot \frac{2\pi}{3}) = \frac{2\pi}{3}$

5.  $\sec^{-1}(\sec \frac{4\pi}{3}) = \frac{4\pi}{3}$

6.  $\csc^{-1}(\csc \frac{5\pi}{6}) = \frac{5\pi}{6}$

7.  $\tan^{-1}(\tan \frac{11\pi}{6}) = \frac{11\pi}{6}$

8.  $\sin^{-1}(\sin \frac{11\pi}{6}) = \frac{11\pi}{6}$

9.  $\cos^{-1}(\cos \frac{11\pi}{6}) = \frac{11\pi}{6}$

10.  $\tan^{-1}(\tan \frac{11\pi}{6}) = \frac{11\pi}{6}$

11.  $\cot^{-1}(\cot \frac{11\pi}{6}) = \frac{11\pi}{6}$

12.  $\sec^{-1}(\sec \frac{11\pi}{6}) = \frac{11\pi}{6}$

13.  $\csc^{-1}(\csc \frac{11\pi}{6}) = \frac{11\pi}{6}$

14.  $\sin^{-1}(\sin \frac{11\pi}{6}) = \frac{11\pi}{6}$

15.  $\cos^{-1}(\cos \frac{11\pi}{6}) = \frac{11\pi}{6}$

16.  $\tan^{-1}(\tan \frac{11\pi}{6}) = \frac{11\pi}{6}$

17.  $\cot^{-1}(\cot \frac{11\pi}{6}) = \frac{11\pi}{6}$

18.  $\sec^{-1}(\sec \frac{11\pi}{6}) = \frac{11\pi}{6}$

19.  $\csc^{-1}(\csc \frac{11\pi}{6}) = \frac{11\pi}{6}$