HW 8.9

Pg. 424 #1-13, 23

1. $y=x^2 \quad \frac{dy}{dx}=2x \quad L=\int_{-1}^{2}\sqrt{1+(2x)^2}\,dx = 6.126$

2. $y=\tan(x) \quad \frac{dy}{dx}=\sec^2(x) \quad L=\int_{-\pi/3}^{0}\sqrt{1+(\sec^2(x))^2}\,dx = 2.057$

3. $x=\sin(y) \quad \frac{dx}{dy}=\cos(y) \quad L=\int_{0}^{\pi}\sqrt{1+\cos^2(y)}\,dy = 3.820$

4. $x=\sqrt{1-y^2} \quad \frac{dx}{dy}=\frac{1}{2}(1-y^2)^{-1/2}(-2y) \quad L=\int_{-1/2}^{1/2}\sqrt{1+[-y(1-y^2)^{-1/2}]^2}\,dy = 1.047$

$= -y(1-y^2)^{-1/2}$

5. $y^2+2y=2x+1$

$y^2+2y-1=2x \quad\quad L=\int_{-1}^{3}\sqrt{1+(y+1)^2}\,dy = 9.294$

$\frac{1}{2}y^2+y-\frac{1}{2}=x \quad \frac{dx}{dy}=y+1$

6. $y=\sin(x)-x\cos(x) \quad\quad L=\int_{0}^{\pi}\sqrt{1+(x\sin(x))^2}\,dx = 4.698$

$\frac{dy}{dx}=\cos(x)-x(-\sin(x))-\cos(x)=x\sin(x)$

7. $y=\int_0^x \tan(t)\,dt \quad L=\int_{0}^{\pi/6}\sqrt{1+\tan^2(x)}\,dx = 0.549$

$\frac{dy}{dx}=\tan(x)$

8. $x=\int_0^y \sqrt{\sec^2(t)-1}\,dt \quad L=\int_{-\pi/3}^{\pi/4}\sqrt{1+(\sec^2(y)-1)}\,dy = 2.198$

$\frac{dx}{dy}=\sqrt{\sec^2(y)-1}$

9. $y=\sec(x) \quad L=\int_{-\pi/3}^{\pi/3}\sqrt{1+\sec^2(x)\tan^2(x)}\,dx = 3.139$

$\frac{dy}{dx}=\sec(x)\tan(x)$

10. $y=\frac{(e^x+e^{-x})}{2} \quad \frac{dy}{dx}=\frac{e^x-e^{-x}}{2} \quad L=\int_{-3}^{3}\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\,dx = 20.036$

11. $y = \frac{1}{3}(x^2+2)^{3/2}$

$\frac{dy}{dx} = \frac{1}{2}(x^2+2)^{1/2} \cdot 2x = (x^2+2)^{1/2}x$

$\left(\frac{dy}{dx}\right)^2 = (x^2+2)x^2 = x^4+2x^2$

$L = \int_0^3 \sqrt{1+x^4+2x^2}\,dx = \int_0^3 \sqrt{x^4+2x^2+1}\,dx = \int_0^3 \sqrt{(x^2+1)^2}\,dx = \int_0^3 x^2+1\,dx$

$= \left(\frac{1}{3}x^3+x\right)\Big|_0^3 = 12$

12. $y = x^{3/2}$ $\quad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$ $\quad \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}x$

$L = \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx = \int_0^4 \left(1+\frac{9}{4}x\right)^{1/2} dx = \left(\frac{2}{3}\left(1+\frac{9}{4}x\right)^{3/2} \cdot \frac{4}{9}\right)\Big|_0^4 = \left(\frac{8}{27}\left(1+\frac{9}{4}x\right)^{3/2}\right)\Big|_0^4$

$= \frac{8}{27}(10)^{3/2} - \frac{8}{27}(1)^{3/2} = \frac{8}{27}(10)^{3/2} - \frac{8}{27}$

13. $x = \frac{y^3}{3} + \frac{1}{4y}$ $\quad \frac{dx}{dy} = y^2 - \frac{1}{4y^2}$ $\quad \left(\frac{dx}{dy}\right)^2 = \left(y^4 - \frac{1}{4} - \frac{1}{4} + \frac{1}{16y^4}\right) = y^4 - \frac{1}{2} + \frac{1}{16y^4}$

$L = \int_1^3 \sqrt{1+y^4-\frac{1}{2}+\frac{1}{16y^4}}\,dy = \int_1^3 \sqrt{y^4+\frac{1}{16y^4}+\frac{1}{2}}\,dy = \int_1^3 \frac{1}{4y^2}\sqrt{16y^8+1+8y^4}\,dy$

$= \int_1^3 \frac{1}{4y^2}\sqrt{16y^8+8y^4+1}\,dy = \int_1^3 \frac{1}{4y^2}\sqrt{(4y^4+1)^2}\,dy = \int_1^3 \frac{1}{4y^2}(4y^4+1)\,dy$

$= \int_1^3 y^2 + \frac{1}{4y^2}\,dy = \int_1^3 y^2 + \frac{1}{4}y^{-2}\,dy = \left(\frac{1}{3}y^3 - \frac{1}{4}y^{-1}\right)\Big|_1^3 = \left(\frac{1}{3}y^3 - \frac{1}{4y}\right)\Big|_1^3$

$= \left(9 - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \left(\frac{108}{12} - \frac{1}{12}\right) - \left(\frac{4}{12} - \frac{3}{12}\right)$

$= \frac{107}{12} - \frac{1}{12} = \frac{106}{12} = \frac{53}{6}$

23. "How wide" becomes length along the curve.

$L = \int_0^{20} \sqrt{1+\left[\frac{3\pi}{20}\cos\left(\frac{3\pi}{20}x\right)\right]^2}\,dx$

$y = \sin\left(\frac{3\pi}{20}x\right)$

$\frac{dy}{dx} = \cos\left(\frac{3\pi}{20}x\right) \cdot \frac{3\pi}{20}$

$L = 21.068\text{ in} = \boxed{21.07\text{ in}}$