Non-Calc Practice

Note: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

1. $x = 3\cos t \quad y = 4\sin t$

We want tangent at $t = 13$. Slope of tangent is $\frac{dy}{dx}$

$\frac{dy}{dt} = 4\cos(t)$ $\quad \frac{dy}{dx} = \frac{4\cos(t)}{-3\sin(t)} = -\frac{4}{3}\cot(t) = -\frac{4}{3}\cdot\frac{1}{\tan(t)} = \frac{-4}{3\tan(t)}$

$\frac{dx}{dt} = -3\sin(t)$

So $\left.\frac{dy}{dx}\right|_{t=13} = \frac{-4}{3\tan(13)}$ (D)

2. Line tangent at $(-3, 8)$ if $x = t^2 - 4t + 1 \quad y = t^3$

Slope: $\frac{dy}{dx}$: $\quad \frac{dy}{dt} = 3t^2 \quad \frac{dx}{dt} = 2t - 4$

$\frac{dy}{dx} = \frac{3t^2}{2t-4}$

We need to know what t to sub in!

well $y = 8$ is given and $x = -3$ is given. Use either to solve for t.

$y = t^3 \rightarrow 8 = t^3 \rightarrow t = 2$

$\frac{dy}{dx} = \frac{3(2)^2}{2(2)-4} = \frac{12}{0} \rightarrow \frac{dy}{dx}$ is undefined, so tangent line is vertical!

vertical line means $x = -3$ (A) b/c point is $(-3, 8)$

3. Line tangent when $t = 1$ if $x = t^2 + 2t$ and $y = t^3 + t^2$

Slope: $\frac{dy}{dx}$: $\quad \frac{dy}{dx} = \frac{3t^2+2t}{2t+2}$ b/c $\frac{dy}{dt} = 3t^2 + 2t$ and $\frac{dx}{dt} = 2t + 2$

if $t = 1 \quad \frac{dy}{dx} = \frac{3+2}{2+2} = \boxed{\frac{5}{4}}$

Point: $x = t^2 + 2t$ at $t = 1$, $x = 1^2 + 2(1) = \boxed{3}$

$y = t^3 + t^2$ at $t = 1$, $y = 1^3 + 1^2 = \boxed{2}$

Equation:

$y - 2 = \frac{5}{4}(x - 3)$

$4y - 8 = 5(x - 3)$

$4y - 8 = 5x - 15$

$4y + 7 = 5x$

$7 = 5x - 4y$ (D)

4. $x = t^2 - 1$ and $y = 2e^t$ $\quad \frac{dy}{dx} = ?$

$\frac{dy}{dt} = 2e^t \quad \frac{dx}{dt} = 2t \quad \frac{dy}{dx} = \frac{2e^t}{2t} = \frac{e^t}{t}$ (A)

5. $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$ $\quad \frac{d^2y}{dx^2} = ?$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ so... $\frac{dy}{dt} = 4t^3 \quad \frac{dx}{dt} = 2t$

$\frac{dy}{dx} = \frac{4t^3}{2t} = 2t^2$ so... $\frac{d}{dt}\frac{dy}{dx} = 4t$

$\frac{d^2y}{dx^2} = \frac{4t}{2t} = 2.$ (B)

Remember the steps: 1. Find dy/dx

2. Divide the derivative of dy/dx by $\frac{dx}{dt}$

6. $x(t) = at^2 \quad y(t) = bt \quad \frac{d^2y}{dx^2} = ? \quad \frac{dy}{dt} = b \quad \frac{dx}{dt} = 2at$

$\frac{dy}{dx} = \frac{b}{2at} = \frac{b}{2a}t^{-1} \qquad \frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{b}{2a}t^{-2} = \frac{-b}{2at^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{-b}{2at^2}}{2at} = \frac{-b}{4a^2t^3}$ (D)