9.4 HW

P.558 #39-51 odd

#39. a) $\langle 2+\int_0^3 3t^2-2t\,dt,\ 6+\int_0^3 1+\cos\pi t\,dt\rangle = \langle 20, 9\rangle$

b) $\int_0^3 \sqrt{(3t^2-2t)^2+(1+\cos\pi t)^2}\,dt = 19.343$

41. a) $\langle 3+\int_0^3 (t+1)^{-1}\,dt,\ -2+\int_0^3 (t+2)^{-2}\,dt\rangle = \langle 3+\ln(4), -1.7\rangle$

b) $\int_0^3 \sqrt{[(t+1)^{-1}]^2+[(t+2)^{-2}]^2}\,dt = 1.419$

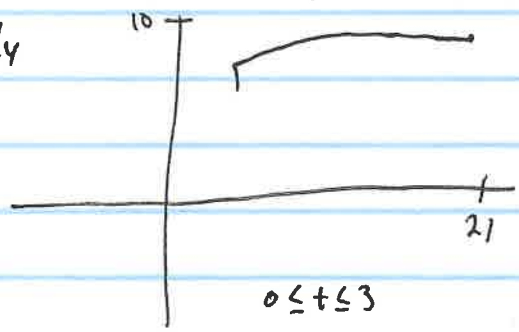
43. $\frac{dx}{dt} = 3t^2-2t$ $\qquad$ $\frac{dy}{dt} = 1+\cos\pi t$

$x = t^3-t^2+C$ when $t=0$ $x=2$ $\qquad$ $y = t+\frac{1}{\pi}\sin\pi t + C$ when $t=0$ $y=6$

$2=C$ $\qquad$ $6=C$

$x=t^3-t^2+2$ $\qquad$ $y = t+\frac{1}{\pi}\sin(\pi t)+6$

Graph parametrically



$0\le t\le 3$

45. $x=5\cos\left(\frac{\pi t}{6}\right)$ $\quad y=3\sin\left(\frac{\pi t}{6}\right)$

a) speed is $|\vec{v}|$

$\frac{dx}{dt} = -\frac{5\pi}{6}\sin\left(\frac{\pi t}{6}\right)$ $\quad \frac{dy}{dt} = \frac{\pi}{2}\cos\left(\frac{\pi t}{6}\right)$

at $t=2$, $\left\langle -\frac{5\pi}{6}\sin\left(\frac{\pi}{3}\right), \frac{\pi}{2}\cos\left(\frac{\pi}{3}\right)\right\rangle = \vec{v}$

$\left\langle \frac{-5\sqrt{3}\pi}{12}, \frac{\pi}{4}\right\rangle = \vec{v}$ so. $|\vec{v}| = \sqrt{\left(\frac{-5\sqrt{3}\pi}{12}\right)^2+\left(\frac{\pi}{4}\right)^2} = 2.399$

b) $\vec{a} = \left\langle \frac{-5\pi^2}{36}\cos\left(\frac{\pi t}{6}\right), -\frac{\pi^2}{12}\sin\left(\frac{\pi t}{6}\right)\right\rangle$

at $t=2$, $\left\langle \frac{-5\pi^2}{72}, \frac{-\sqrt{3}\pi^2}{24}\right\rangle$

c) $\left(\frac{5\cos(\pi t/6)}{5}\right)^2 + \left(\frac{3\sin(\pi t/6)}{3}\right)^2 = 1$

$\left(\frac{x}{5}\right)^2+\left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{25}+\frac{y^2}{9} = 1$

47. a) $\vec{v} = \left\langle \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2}, \frac{(1+t^2)(2)-(2t)(2t)}{(1+t^2)^2} \right\rangle$

$\vec{v} = \left\langle \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}, \frac{2+2t^2-4t^2}{(1+t^2)^2} \right\rangle = \left\langle \frac{-4t}{(1+t^2)^2}, \frac{2-2t^2}{(1+t^2)^2} \right\rangle$

b) Particle at rest if $|\vec{v}|$ is 0. Need both x and y-component of velocity to be 0.

$\frac{dx}{dt} = \frac{-4t}{(1+t^2)^2}$ At $t=0$, $\frac{dx}{dt} = 0$

but at $t=0$, $\frac{dy}{dt} = 2$, so particle is never at rest. Velocity never $\langle 0, 0 \rangle$

c) Sounds like a limit.

$\lim_{t\to\infty} \vec{p} = \left\langle \lim_{t\to\infty} \frac{1-t^2}{1+t^2}, \lim_{t\to\infty} \frac{2t}{1+t^2} \right\rangle = \langle -1, 0 \rangle$

$\hookrightarrow$ EBM is $\frac{-t^2}{t^2} = -1$ $\hookrightarrow$ EBM is $\frac{2t}{t^2} = \frac{2}{t}$ as $t \to \infty$

49. a) Slope is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{dy}{dt} = e^t(-\sin(t)) + e^t\cos(t) = -e^t\sin(t) + e^t\cos(t)$

$\frac{dx}{dt} = e^t\cos(t) + e^t\sin(t)$

$\frac{dy}{dx} = \frac{-e^t\sin(t)+e^t\cos(t)}{e^t\cos(t)+e^t\sin(t)}$ when $t = \frac{\pi}{2}$ $\frac{dy}{dx} = \frac{-e^{\pi/2}}{e^{\pi/2}} = -1$

b) $\left.\frac{dy}{dt}\right|_{t=1} = -0.81866$ $\left.\frac{dx}{dt}\right|_{t=1} = 3.75605$

Speed $= \sqrt{(3.75605)^2 + (-0.81866)^2} = 3.844$

c) $L = \int_0^1 \sqrt{(e^t\cos t + e^t\sin t)^2 + (-e^t\sin t + e^t\cos t)^2}\, dt = 2.430$

51. a) x-coordinate is $3 + \int_2^4 2+\sin(t^2)\, dt = 3.942$

x-coord at $t=2$

b) tangent line:

Slope: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-6}{2+\sin(4)}$

point: $(3, 5)$ so... $y - 5 = \frac{-6}{2+\sin(4)}(x-3)$

c) speed $= \sqrt{(2+\sin(4))^2 + (-6)^2} = 6.127$

d) $\frac{dy}{dx} = 2t-1$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ so. $2t-1 = \frac{\frac{dy}{dt}}{2+\sin(t^2)}$

$(2t-1)(2+\sin(t^2)) = \frac{dy}{dt}$ and $\frac{dx}{dt} = 2+\sin(t^2)$

$\vec{a} = \langle 2t\cos(t^2), (2t-1)(2t\cos(t^2)) + (2)(2+\sin(t^2)) \rangle$

at $t=4$ $\vec{a} = \langle 8\cos(16), 7(8\cos(16)) + 2(2+\sin(16)) \rangle$

$\vec{a} = \langle 8\cos(16), 56\cos(16) + 4 + 2\sin(16) \rangle = \langle -7.661, -50.205 \rangle$