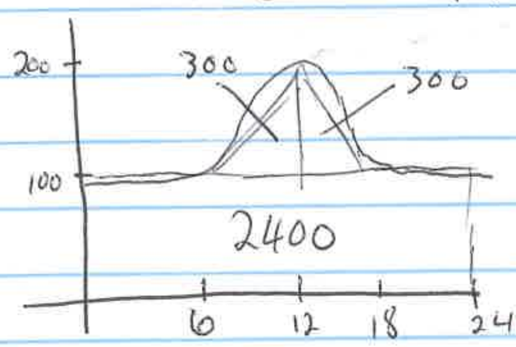
Unit 6 Review Problems

1. $\int_0^{24} \text{Rate of flow } dt$ = total amount of oil in barrels.

Area under curve = 3000 barrels (D)



2. $\int_0^2 f(x)dx = \int_0^2 9^x dx$ four equal sub intervals Right Sum

x	0	1/2	1	3/2	2
f(x)	1	3	9	27	81

$\int_0^2 9^x dx \approx \frac{1}{2}(3)+\frac{1}{2}(9)+\frac{1}{2}(27)+\frac{1}{2}(81) = \frac{120}{2} = 60$ (C)

$\Delta x f(x_1) + \Delta x f(x_2)$

3. For a decreasing and CCD curve, LRAM over approximates, RRAM under approximates, TRAP under approximates but not as much, and Midpoint is between L and RRAM

LRAM RRAM

(C)

Note $\int_1^3 f(x)\,dx$ is the <u>exact</u> amount under curve

4. $\int_2^{13} f(x)dx \approx 1(6)+2(-2)+3(-1)+5(3) = 6-4-3+15 = 14$ (B)

Δx changes each interval.

5. $\int_2^5 x^2 dx \Rightarrow \lim_{x\to\infty} \sum_{k=1}^{n} f(a+\Delta x k)\Delta x \Rightarrow \lim_{x\to\infty} \sum_{k=1}^{n} \left(2+\frac{3k}{n}\right)^2 \frac{3}{n}$ (D)

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$

6. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{12k}{n} \cos\left(1+\frac{4k}{n}\right)\frac{4}{n} = \int_0^4 3x\cos(1+x)\,dx$ (D)

$\Delta x = \frac{4}{n}$ so $b-a=4$

1 can't be "a", b/c it would appear in front of cos( ), too

Plug in $\sqrt{x}$ for t

deriv of $\sqrt{x}$

7. $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{1}{1+t^2}\,dt = \frac{1}{1+x}\cdot\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$ A

8. $f(x)=\int_{10}^{x}(-t^2+2t+3)\,dt$

$f(x)$ is inc when $f'(x)$ is $+$

$f'(x) = -x^2+2x+3 = -1(x^2-2x-3) = -1(x-3)(x+1)$

	-1		3	
	$-$		$+$	$-$
-1	$-$		$-$	$-$
$(x-3)$	$-$		$-$	$+$
$(x+1)$	$-$		$+$	$+$

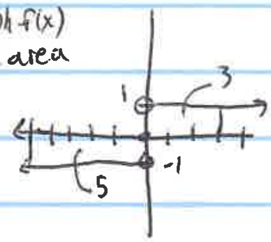
$f(x)$ is inc $(-1,3)$ b/c $f'(x)$ is $+$ on that interval

B

9. $g(x)=\int_1^x f(t)\,dt$ g has rel. max when $g'(x)$ changes $+\to-$. $g'(x)=f(x)$

At $x=1$, $g'(x)=f(x)$ changes $+\to-$. A

10. graph $f(x)$ find area



$\int_{-5}^{3} f(x)\,dx = -5+3 = -2$ A

11. $\int_0^7 f(x)\,dx = 1+2+4+1-2 = 6$ A

Find area

12. $\int_0^4 f(x)\,dx = 2+4-6 = 0$ B

Find area

13. $\int_{-2}^{8}(3g(x)+2)\,dx = 35$

$3\int_{-2}^{8} g(x)\,dx + \int_{-2}^{8} 2\,dx = 35$

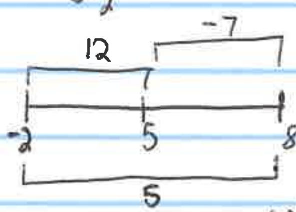
$3\int_{-2}^{8} g(x)\,dx + 20 = 35$

$3\int_{-2}^{8} g(x)\,dx = 15$

$\int_{-2}^{8} g(x)\,dx = 5$

$\int_5^{-2} g(x)\,dx = -12$

so $\int_{-2}^{5} g(x)\,dx = 12$



$\int_5^8 g(x)\,dx = -7$ B

14. $g(3) = g(0) + \int_0^3 g'(x)\,dx = 1 + \int_0^3 g'(x)\,dx = 1 + \frac{\pi(2)^2}{4} + 1 = 2+\pi$ B

Accumulation function

15. $\int_0^{\pi/2} \frac{\cos(\theta)}{\sqrt{1+\sin(\theta)}}\,d\theta = \int_1^2 \frac{\cos(\theta)}{\sqrt{u}} \frac{1}{\cos(\theta)}\,du = \int_1^2 \frac{1}{\sqrt{u}}\,du = \int_1^2 u^{-1/2}\,du$

$= 2u^{1/2}\Big|_1^2$

$u = 1+\sin(\theta)$ $\quad x=\pi/2 \rightarrow u=2$ $\quad = 2\sqrt{2} - 2\sqrt{1}$

$\frac{du}{dx} = \cos(\theta)$ $\quad x=0 \rightarrow u=1$ $\quad = 2\sqrt{2} - 2 = 2(\sqrt{2}-1)$ D

$\frac{1}{\cos(\theta)}\,du = dx$

16. $\int_0^1 (3x-2)^2\,dx = \int_{-2}^1 u^2 \frac{1}{3}\,du = \int_{-2}^1 \frac{1}{3}u^2\,du = \left(\frac{1}{9}u^3\right)\Big|_{-2}^1 = \frac{1}{9} - \left(\frac{-8}{9}\right) = 1$ D

$u = 3x-2$ $\quad x=1 \rightarrow u=1$, $x=0 \rightarrow u=-2$

$\frac{du}{dx} = 3$

$\frac{1}{3}\,du = dx$

17. $\int_1^4 |x-3|\,dx = 2.5 = \frac{5}{2}$ C

18. $\int \frac{x^3+5}{x^2}\,dx = \int x + \frac{5}{x^2}\,dx = \int x + 5x^{-2}\,dx = \frac{1}{2}x^2 - 5x^{-1} + C = \frac{x^2}{2} - \frac{5}{x} + C$ C

$x^2 \overline{)\,x^3+5}$ quotient x

$-x^3$

$0+5$

19. $\int \sin(2x) + \cos(2x)\,dx = -\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$ B

fast u-sub

20. $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^2 \frac{e^u}{\sqrt{x}} 2\sqrt{x}\,du = \int_1^2 2e^u\,du = 2\int_1^2 e^u\,du$ C

$x=4 \rightarrow u=2$

$u = \sqrt{x}$ $\quad x=1 \rightarrow u=1$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\,du = dx$

21. $\int_1^2 \frac{x^2-x-5}{x+2}\,dx = \int_1^2 x - 3 + \frac{1}{x+2}\,dx = \left(\frac{x^2}{2} - 3x + \ln|x+2|\right)\Big|_1^2$

$= (2 - 6 + \ln|4|) - \left(\frac{1}{2} - 3 + \ln|3|\right)$

$= -4 + \ln|4| - \frac{1}{2} + 3 - \ln|3| = \frac{-3}{2} + \ln|4| - \ln|3|$

$= -\frac{3}{2} + \ln(4/3)$ A

$x+2 \overline{)\,x^2-x-5}$ quotient $x-3$

$-(x^2+2x)$

$-3x-5$

$-(-3x-6)$

1

22. $\int \frac{1}{x^2+4x+5}\,dx = \int \frac{1}{(x^2+4x)+5}\,dx = \int \frac{1}{(x^2+4x+4)+5-4}\,dx = \int \frac{1}{(x+2)^2+1}\,dx$

Complete the square

$= \arctan(x+2)+C$

A

23. $\int x f(x)\,dx$ $\quad u=f(x) \quad dv = x\,dx$

Int by parts $\quad du = f'(x)\,dx \quad v=\frac{x^2}{2}$

$\int x f(x)\,dx = \frac{x^2}{2}f(x) - \int \frac{x^2}{2} f'(x)\,dx$ $\quad$ B

24. $\int_1^e x^4 \ln(x)\,dx$ $\quad u=\ln(x) \quad dv = x^4\,dx$

$du = \frac{1}{x}dx \quad v=\frac{x^5}{5}$

$\int_1^e x^4\ln(x)\,dx = \ln(x)\frac{x^5}{5}\Big|_1^e - \int_1^e \frac{x^5}{5}\cdot\frac{1}{x}\,dx = \frac{x^5}{5}\ln(x)\Big|_1^e - \int_1^e \frac{x^4}{5}\,dx = \frac{x^5}{5}\ln(x)\Big|_1^e - \frac{x^5}{25}\Big|_1^e$

$= \left(\frac{x^5}{5}\ln(x) - \frac{x^5}{25}\right)\Big|_1^e = \left(\frac{e^5}{5}\ln(e) - \frac{e^5}{25}\right) - \left(\frac{1}{5}\ln(1) - \frac{1}{25}\right)$

$= \frac{e^5}{5} - \frac{e^5}{25} + \frac{1}{25} = \frac{4e^5}{25} + \frac{1}{25} = \frac{4e^5+1}{25}$

B

25. $\int \frac{7x}{(2x-3)(x+2)}\,dx = \int \frac{A}{2x-3} + \frac{B}{x+2}\,dx = \int \frac{3}{2x-3} + \frac{2}{x+2}\,dx$

$= 3\cdot\frac{1}{2}\ln|2x-3| + 2\ln|x+2| + C$

$7x = A(x+2) + B(2x-3)$ $\quad$ fast u-sub $\quad = \frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

A

$x=-2 \longrightarrow -14 = B(-4-3)$

$B=2$

$x=\frac{3}{2} \longrightarrow \frac{21}{2} = A\left(\frac{7}{2}\right)$

$A=3$

26. $\int \frac{1}{x^2-7x+10}\,dx = \int \frac{1}{(x-2)(x-5)}\,dx = \int \frac{A}{x-2} + \frac{B}{x-5}\,dx = \int \frac{-\frac{1}{3}}{x-2} + \frac{\frac{1}{3}}{x-5}\,dx$

$1 = A(x-5) + B(x-2)$ $\quad = -\frac{1}{3}\ln|x-2| + \frac{1}{3}\ln|x-5| + C$

$x=5 \rightarrow 1 = B(3) \rightarrow B=\frac{1}{3}$ $\quad = \frac{1}{3}\ln|x-5| - \frac{1}{3}\ln|x-2| + C$

$x=2 \rightarrow 1 = A(-3) \rightarrow A = -\frac{1}{3}$ $\quad = \frac{1}{3}\left[\ln|x-5| - \ln|x-2|\right] + C$

$= \frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + C$ $\quad$ E

27. $\int_1^\infty \frac{x^2}{(x^3+2)^2}\,dx = \lim_{a\to\infty}\int_1^a \frac{x^2}{(x^3+2)^2}\,dx = \lim_{a\to\infty}\int_3^{a^3+2} \frac{x^2}{u^2}\frac{1}{3x^2}\,du = \lim_{a\to\infty}\int_3^{a^3+2} \frac{1}{3}u^{-2}\,du$

$u = x^3+2$ $\quad x=a \rightarrow u=a^3+2$ $\quad x=1 \rightarrow u=3$

$\frac{du}{dx} = 3x^2$

$\frac{1}{3x^2}\,du = dx$

$= \lim_{a\to\infty}\left(-\frac{1}{3}u^{-1}\right)\Big|_3^{a^3+2} = \lim_{a\to\infty}\left[\left(-\frac{1}{3(a^3+2)}\right)-\left(-\frac{1}{9}\right)\right]$

$= \frac{1}{9}$ (B)

28. $\int_0^3 \frac{dx}{(1-x)^2} = \lim_{a\to1^-}\int_0^a \frac{1}{(1-x)^2}\,dx + \lim_{b\to1^+}\int_b^3 \frac{1}{(1-x)^2}\,dx = \lim_{a\to1^-}\int_0^a (1-x)^{-2}\,dx + \lim_{b\to1^+}\int_b^3 (1-x)^{-1}\,dx$

inf. discontinuity at $x=1$

$= \lim_{a\to1^-}(1-x)^{-1}\Big|_0^a + \lim_{b\to1^+}(1-x)^{-1}\Big|_b^3$

fast u-sub ↗ $\quad = \lim_{a\to1^-}\left[(1-a)^{-1}-(1)^{-1}\right] + \lim_{b\to1^+}\left[(-2)^{-1}-(1-b)^{-1}\right]$

$\frac{1}{0}$ $\qquad \frac{1}{0}$

Diverges (E)

29. $\int_1^2 \frac{x^7-4x^3+6}{x^2}\,dx = \int_1^2 x^5-4x+\frac{6}{x^2}\,dx = \int_1^2 x^5-4x+6x^{-2}\,dx$ (C)