Pg. 534 #18-21 10.5 HW

#18. $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln(n)}$ try the alternating series test (AST)

1. Is $\frac{1}{\ln(n)} > 0$ for $n \geq 2$? yes! ✓
2. Is $\frac{1}{\ln(n)} > \frac{1}{\ln(n+1)}$? yes ✓
3. $\lim_{n\to\infty} \frac{1}{\ln(n)} = 0$ ✓

Therefore by the alternating series test, $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln(n)}$ converges

#19 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$ try AST

1. $\frac{10^n}{n^{10}} > 0$ ✓
2. $\frac{10^n}{n^{10}} \not> \frac{10^{n+1}}{(n+1)^{10}}$ b/c 10^n grows faster than $(n)^{10}$ X try different test

Try n^{th} term test $\lim_{n\to\infty} (-1)^{n+1} \frac{10^n}{n^{10}} \Rightarrow$ DNE (10^n grows faster than n^{10}) so $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$ diverges by the n^{th} term test for divergence

#20. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$ try AST

1. $\frac{\sqrt{n}+1}{n+1} > 0$ ✓
2. $\frac{\sqrt{n}+1}{n+1} > \frac{\sqrt{n+1}+1}{n+2}$ ✓ b/c n will grow faster than $\sqrt{n+1}$, fraction will eventually get smaller
3. $\lim_{n\to\infty} \frac{\sqrt{n}+1}{n+1} \overset{H}{=} \lim_{n\to\infty} \frac{\frac{1}{2}n^{-1/2}}{1} = \lim_{n\to\infty} \frac{1}{2\sqrt{n}} = 0$ ✓

Therefore $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$ converges by the alternating series test.

#21 $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{\ln(n^2)}$ try AST

1. $\frac{\ln(n)}{\ln(n^2)} > 0$ ✓
2. $\frac{\ln(n)}{\ln(n^2)} > \frac{\ln(n+1)}{\ln([n+1]^2)}$ this is curious! Use log properties!

$$\frac{\ln(n)}{2\ln(n)} > \frac{\ln(n+1)}{2\ln(n+1)}$$

$$\frac{1}{2} \not> \frac{1}{2}$$ so AST does not apply!

Try n^{th} term test $\lim_{n\to\infty} (-1)^{n+1} \frac{\ln(n)}{\ln(n^2)} = \lim_{n\to\infty} (-1)^{n+1} \frac{\ln(n)}{2\ln(n)} = \lim_{n\to\infty} (-1)^{n+1} \cdot \frac{1}{2} \Rightarrow$ DNE keeps alternating!

$\therefore \sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{\ln(n^2)}$ diverges by the n^{th} term test for divergence.