10.8 HW

1. $f(x) = \ln(x) \quad a = 1$

0 term: $f(1) = 0$ $\quad f'(x) = \frac{1}{x} = x^{-1}$

Non-0 terms:

$f'(1) = 1 \quad f''(x) = -x^{-2}$

$f''(1) = -1 \quad f'''(x) = 2x^{-3}$

$f'''(1) = 2 \quad f^{(4)}(x) = -6x^{-4}$

$f^{(4)}(1) = -6$

$$T(x) = \frac{0}{0!}(x-1)^0 + \frac{1}{1!}(x-1)^1 + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4 + \dots$$

$$T(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{6}{4!}(x-1)^4 + \dots$$

$$\star\ T(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots + (-1)^{n+1}\frac{1}{n}(x-1)^n + \dots$$

assume n starts at 0.

2. Maclaurin Series is Taylor series centered at $x = 0$

a) $f(x) = \sin(2x) \quad a = 0$

$f(0) = \sin(0) = 0 \quad f'(x) = 2\cos(2x)$

$f'(0) = 2\cos(0) = 2 \quad f''(x) = -4\sin(2x)$

$f''(0) = -4\sin(0) = 0 \quad f'''(x) = -8\cos(2x)$

$f'''(0) = -8\cos(0) = -8 \quad f^{(4)} = 16\sin(2x)$

$f^{(4)}(0) = 16\sin(0) = 0 \quad f^{(5)} = 32\cos(2x)$

$f^{(5)}(0) = 32\cos(0) = 32$

$$T(x) = \frac{0}{0!}x^0 + \frac{2}{1!}x^1 + \frac{0}{2!}x^2 + \frac{-8}{3!}x^3 + \frac{0}{4!}x^4 + \frac{32}{5!}x^5 + \dots$$

$$\star\ T(x) = \underset{n=0}{2x} - \underset{n=1}{\frac{8}{3!}x^3} + \underset{n=2}{\frac{32}{5!}x^5} + \dots + (-1)^n \frac{2^{2n+1}}{(2n+1)!}x^{2n+1} + \dots$$

b) yes, we know $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

↳ memorize this.

so we could plug $2x$ in for x is this $\sin(x)$ series!

3. $f(1) = 1 \quad f'(x) = \frac{1}{2}x^{-1/2} \quad f(x) = \sqrt{x} \quad a = 1$

$f'(1) = \frac{1}{2} \quad f''(x) = -\frac{1}{4}x^{-3/2} \quad f(x) = x^{1/2}$

$f''(1) = -\frac{1}{4} \quad f'''(x) = +\frac{3}{8}x^{-5/2}$

$f'''(1) = +\frac{3}{8} \quad f^{(4)}(x) =$ oh wait, we're done!

4 non-zero terms!

$$P_3(x) = \frac{1}{0!}(x-1)^0 + \frac{\frac{1}{2}}{1!}(x-1)^1 + \frac{-\frac{1}{4}}{2!}(x-1)^2 + \frac{\frac{3}{8}}{3!}(x-1)^3$$

$$P_3(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$$

$$\star\ T(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 + \dots$$

4. a) $f(x)=2^x \quad a=2$

$f(2)=2^2=4 \qquad f'(x)=\ln(2)2^x$

$f'(2)=\ln(2)4 \qquad f''(x)=\ln^2(2)\cdot 2^x$

$f''(2)=\ln^2(2)4 \qquad f'''(x)=\ln^3(2)\cdot 2^x$

$f'''(2)=\ln^3(2)\cdot 4$

$$T(x)=\frac{4(x-2)^0}{0!}+\frac{\ln(2)4}{1!}(x-2)^1+\frac{\ln^2(2)4}{2!}(x-2)^2+\frac{\ln^3(2)4}{3!}(x-2)^3+\dots\frac{4\ln^n(2)}{n!}(x-2)^n+\dots$$

$$T(x)=4+\ln(2)4(x-2)+\ln^2(2)2(x-2)^2+\frac{\ln^3(2)2}{3}(x-2)^3+\dots+\frac{4\ln^n(2)}{n!}(x-2)^n+\dots$$

5. a) $f(x)=? \quad a=2$

$f(2)=1 \qquad f'(2)=\frac{(1+1)!}{3^1}=\frac{2}{3} \qquad f''(2)=\frac{(2+1)!}{3^2}=\frac{3!}{9}=\frac{2}{3} \qquad f'''(2)=\frac{(3+1)!}{3^3}=\frac{4!}{3^3}=\frac{8}{9}$

$$T(x)=\frac{1}{0!}(x-2)^0+\frac{\frac{2}{3}}{1!}(x-2)^1+\frac{\frac{2}{3}}{2!}(x-2)^2+\frac{\frac{8}{9}}{3!}(x-2)^3+\dots+\frac{\frac{(n+1)!}{3^n}}{n!}(x-2)^n+\dots$$

$$T(x)=1+\frac{2}{3}(x-2)^1+\frac{1}{3}(x-2)^2+\frac{4}{27}(x-2)^3+\dots+\frac{\frac{(n+1)!}{3^n}}{n!}(x-2)^n+\dots$$

↳ could simplify to $\frac{n+1}{3^n}(x-2)^n$

b) $g(x)=? \quad a=2 \quad g'(x)=f(x)$

$g(2)=3 \qquad g'(x)=f(x)$

$g'(2)=f(2)=1 \qquad g''(x)=f'(x) \Rightarrow f'(2)=$____

$g''(2)=f'(2)=\frac{(1+1)!}{3^1}=\frac{2}{3} \qquad g'''(x)=f''(x) \Rightarrow f''(2)=$____

$g'''(2)=f''(2)=\frac{(2+1)!}{3^2}=\frac{2}{3}$

$$T_g(x)=\frac{3}{0!}(x-2)^0+\frac{1}{1!}(x-2)^1+\frac{\frac{2}{3}}{2!}(x-2)^2+\frac{\frac{2}{3}}{3!}(x-2)^3+\dots+\frac{\frac{n!}{3^{n-1}}}{n!}(x-2)^n+\dots$$

↓ if it is too hard too see this, simplify the Taylor series.

$$T_g(x)=3+(x-2)+\frac{1}{3}(x-2)^2+\frac{1}{9}(x-2)^3+\dots+\frac{1}{3^{n-1}}(x-2)^n+\dots$$

That helps sometimes