HW 2.6

Pg. 116 #1-10, 31-35 odd, 36, 39 pg. 151 #57-58, 63

Pg. 116 #1 L: $\frac{dx^2}{dx} = 2x$ at $x=0$ $2(0)=\boxed{0}$

R: $\frac{d}{dx}x = 1$ at $x=0$ $\boxed{1}$

$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} \neq \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$

$0 \neq 1$

#2 L: $\frac{d}{dx}2 = 0$ at $x=1$ $\boxed{0}$

R: $\frac{d}{dx}2x = 2$ at $x=1$ $\boxed{2}$

$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} \neq \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

$0 \neq 2$

#3 L: $\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-1/2}$ at $x=1$ $\frac{1}{2}(1)^{-1/2} = \boxed{\frac{1}{2}}$

R: $\frac{d}{dx}(2x-1) = 2$ at $x=1$ $\boxed{2}$

$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} \neq \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

$\frac{1}{2} \neq 2$

#4 L: $\frac{d}{dx}x = 1$ at $x=1$ $\boxed{1}$

R: $\frac{d}{dx}\frac{1}{x} = -x^{-2}$ at $x=1$ $\boxed{-1}$

$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} \neq \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

$1 \neq -1$

#5 a) differentiable $[-3,2]$

b) Nowhere, cont but not diff

c) Nowhere, neither cont nor diff

#6 a) differentiable $[-2,3]$

b) Nowhere, cont but not diff

c) Nowhere, neither continuous nor diff

#7 a) $[-3,3]$ except $x=0$

b) Nowhere

c) At $x=0$

#8 a) $[-2,3]$ except $x=-1,0,2$

b) at $x=-1$

c) at $x=0,2$

#9 a) $[-1,2]$ except $x=0$

b) at $x=0$

c) Nowhere

#10 a) $[-3,3]$ except $x=-2,2$

b) $x=-2,2$

c) Nowhere

#31 $f(x) = \frac{x^3-8}{(x-5)(x+1)}$ $\boxed{\mathbb{R}, f(x) \text{ is not differentiable at } x=5,-1 \text{ (discontinuities)}}$

#32 $h(x) = \sqrt[3]{3x-6}+5$ $\boxed{\mathbb{R}, h(x) \text{ is not differentiable at } x=2 \text{ (vertical tangent)}}$

Note: $\sqrt[3]{\ }$ have vertical tangents when what's under $\sqrt[3]{\ }$ is 0.

#33 $P(x) = \sin(|x|)-1$ $\boxed{P(x) \text{ is not differentiable at } x=0 \text{ (cusp), otherwise } \mathbb{R}}$

#34 $Q(x) = 3\cos(|x|)$ $\boxed{\mathbb{R}\ Q(x) \text{ is always differentiable; cosine is an even function}}$, so $3\cos(|x|) = 3\cos(x)$. Losing the (-) doesn't matter

#35 $g(x)=\begin{cases}(x+1)^2, & x\le 0\\ 2x+1, & 0<x<3\\ (4-x)^2, & x\ge 3\end{cases} = \begin{cases}x^2+2x+1, & x\le 0\\ 2x+1, & 0<x<3\\ 16-8x+x^2, & x\ge 3\end{cases}$

at $x=0$ check continuity and differentiability

Cont: $x^2+2x+1=2x+1$ at $x=0$ $\quad 1=1$ ✓

Diff: $2x+2=2$ at $x=0$ $\quad 2=2$ ✓

at $x=3$

cont: $2x+1=16-8x+x^2$ at $x=3$

$7=1$ ✗

$g(x)$ is not differentiable at $x=3$ (discontinuity), otherwise $\mathbb{R}$

#36 $c(x)=x|x|$ $\boxed{\text{c(x) Is always differentiable, } \mathbb{R}}$

This is due to the x in front of the $|x|$

#39 $f(x)=\begin{cases}3-x, & x<1\\ ax^2+bx, & x\ge 1\end{cases}$

a) Cont: $3-x=ax^2+bx$ at $x=1$

$2=a+b$ or $2-a=b$

b) Diff: $-1=2ax+b$ at $x=1$

$-1=2a+b$ or $-1-2a=b$

$2-a=-1-2a$

$3=-a$

$a=-3$ so $b=5$

Pg 151 #57 $f(x)=\begin{cases}2x-3, & -1\le x<0\\ x-3, & 0\le x\le 4\end{cases}$

cont: $2x-3=x-3$ at $x=0$

$-3=-3$ continuous

diff: $2\ne 1$ at $x=0$

Not differentiable at $x=0$

a) $[-1,4]$ except $x=0$

b) at $x=0$

c) Nowhere

#58 $g(x)=\begin{cases}\frac{x-1}{x}, & -2\le x<0\\ \frac{x+1}{x}, & 0\le x\le 2\end{cases}$

Cont: $\frac{x-1}{x}=\frac{x+1}{x}$ at $x=0$

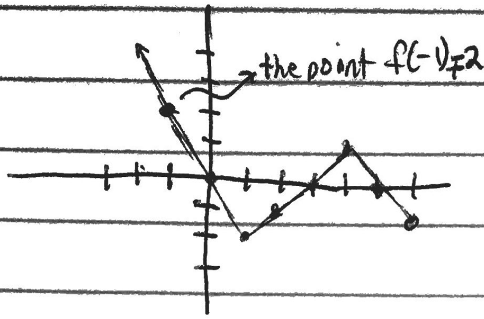
Neither side exists: Vertical asymptotes

diff: $\frac{(x)(1)-(x-1)(1)}{x^2}=\frac{x(1)-(x+1)(1)}{x^2}$ at $x=0$

Derivatives DNE

a) $[-2,2]$ except $x=0$ $\quad$ b) Nowhere $\quad$ c) at $x=0$

#63



Rememember, f'(x) measures slope!