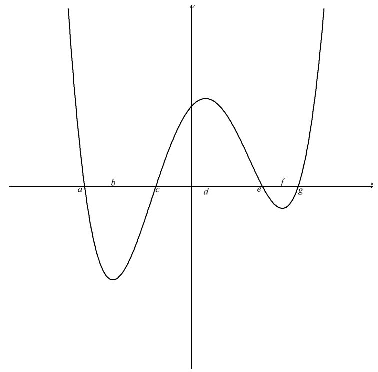
AP Calculus BC Practice Exam

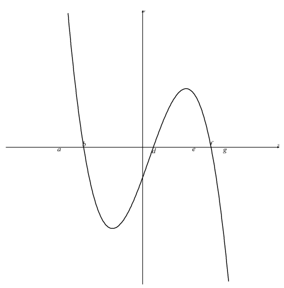
CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—27
A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

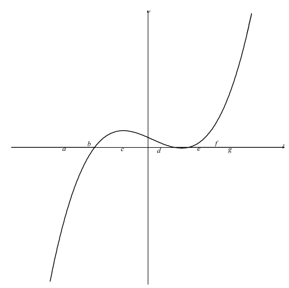
Directions: Solve each of the following problems. After examining the choices, select the choice that best answers the question. No credit will be given for anything written in the test book.

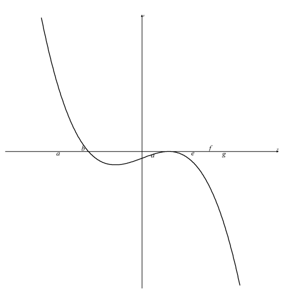
In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

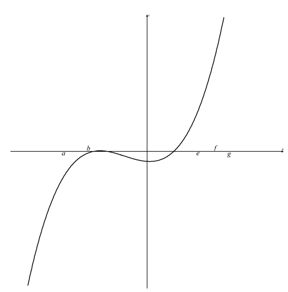


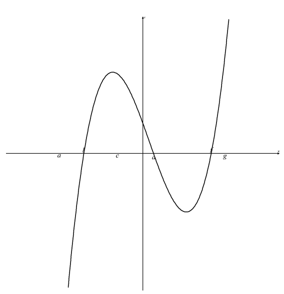
1. The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f'(x)$?

A. 

D. 

B. 

E. 

C. 

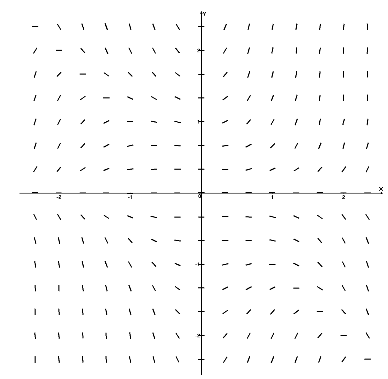
2. Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative minimum at $x = c$ and $a < c < b$, which of the following statements must be true?

I. $f''(c) = 0$.

II. $f'(c) < 0$.

III. f is not differentiable at $x = c$.

A. I only
B. II only
C. III only
D. All of these
E. None of these



3. Shown above is the slope field for which of the following differential equations?

A. $\frac{dy}{dx} = x^2 y$

B. $\frac{dy}{dx} = xy^2$

C. $\frac{dy}{dx} = xy + xy^2$

D. $\frac{dy}{dx} = x^2 y + xy$

E. $\frac{dy}{dx} = x^2 y + xy^2$

4. $\int \frac{1}{x^2 - 7x - 18} dx =$

A. $\ln(x^2 - 7x - 18)$

B. $\ln(x-9) + \ln(x+2)$

C. $\ln(x-9) - \ln(x+2)$

D. $\frac{1}{11}[\ln(x-9) - \ln(x+2)]$

E. $\frac{2}{9}[\ln(x-9) - \ln(x+2)]$

5. $\int x^2 \sin x dx =$

A. $-x^2 \cos x + 2x \sin x + 2\cos x + C$

B. $-x^2 \cos x + 2x \sin x - 2\cos x + C$

C. $-x^2 \cos x + 2x \sin x + 2x \cos x + C$

D. $x^2 \cos x - 2x \sin x + 2x \cos x + C$

E. $x^2 \cos x + 2x \sin x + 2x \cos x + C$

6. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^2 - t^3$ and its y-coordinate is $(2-3t)^2$. The acceleration vector of the particle at $t = 2$ is

A. $\langle -8, 24 \rangle$

B. $\langle -10, 18 \rangle$

C. $\langle -4, 16 \rangle$

D. $\langle 0, -12 \rangle$

E. $\langle 2, 18 \rangle$

7. If $\frac{dy}{dx} = \cos x \sin^2 x$ and if $y = 0$ when $x = 0$, what is the value of y when $x = \frac{\pi}{2}$?

A. 1

B. $\frac{1}{3}$

C. 0

D. $-\frac{1}{3}$

E. -1

8. If f is the function defined by $f(x) = 2x^6 - 9x^4$, what are all the x-coordinates of relative extrema of the graph of f?

A. 0

B. $\sqrt{3}$

C. $-\sqrt{3}$

D. All of these

E. None of these

9. If f is twice differentiable and if $g(x) = f(f(x))$, then $g''(x) =$

A. $f'(f(x))f'(x)$

B. $f'(f(x))f''(x)$

C. $[f'(x)]^2 f''(f(x))$

D. $f'(f(x))f'(x) + f(x)f''(f(x))$

E. $f'(f(x))f''(x) + [f'(x)]^2 f''(f(x))$

10. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f''(2)$

A. $\sum_{n=1}^{\infty} n a_n 2^{n-1}$

B. $\sum_{n=1}^{\infty} (n-1) a_n 2^{n-2}$

C. $\sum_{n=2}^{\infty} n(n-1) a_n 2^{n-2}$

D. $\sum_{n=0}^{\infty} n a_n 2^{n-1}$

E. $\sum_{n=2}^{\infty} n a_n 2^{n-2}$

11. If $\lim\limits_{k\to\infty}\int_1^k \frac{dx}{x^p}$ is finite, then which of the following must be true?

I. $\sum\limits_{n=1}^{\infty}\frac{1}{x^{-p}}$ converges

II. $\sum\limits_{n=1}^{\infty}\frac{1}{x^{|p|}}$ diverges

III. $\sum\limits_{n=1}^{\infty}\frac{1}{x^{p^2}}$ converges

A. I only
B. II only
C. III only
D. I and III
E. II and III

12. In the xy-plane, the graph of the parametric equations $x = t + 3$ and $y = (t - 2)^2$, for $-4 \le t \le 4$ is a parabola with y-intercept

A. 25
B. 16
C. 4
D. –5
E. –2

13. $\int_0^{\infty} x^3 e^{2x^4}\,dx$ is

A. 0
B. e^2
C. $\frac{1}{8}$
D. $\frac{e}{8}$
E. divergent

14. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(500 - \frac{P}{4}\right)$, where the initial population $P(0) = 100$ and t is the time in years. What is $\lim\limits_{t\to\infty} P(t)$?

A. 19
B. 500
C. 1900
D. 2000
E. 38000

15. What are all values of x for which the function defined by $f(x) = 3x^2 - 4x$ is decreasing?

A. $x < -\frac{2}{3}$
B. $x > -\frac{2}{3}$
C. $-\frac{2}{3} < x < \frac{2}{3}$
D. $x < \frac{2}{3}$
E. $x > \frac{2}{3}$

16. When $x = 3$, the rate at which $\frac{1}{x}$ is decreasing is k times the rate at which x is increasing. What is the value of k?

A. 9
B. 3
C. $\frac{1}{9}$
D. $-\frac{1}{9}$
E. -3

17. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n+1}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{\sin(n\pi)}{n}$

III. $\sum_{n=1}^{\infty} \frac{2}{n+1}$

A. I only
B. II only
C. III only
D. I and II
E. II and III

18. For what value of k will $\frac{x^2-k}{x-1}$ have a relative minimum at $x = 3$?

A. $k = -3$
B. $k = -1$
C. $k = 0$
D. $k = 1$
E. $k = 3$

19. If $h(x) = f(x)g(x)(1 + g(x))$ and $g(1) = 0$, then $h'(1) =$

A. $f'(1) + g'(1)$
B. $f'(1) + g'(1)f(1)$
C. $f(1)g'(1)$
D. $f'(1)g'(1) + f(1)$
E. $f'(1)g'(1) + f'(1)$

20. If $\ln y + \cos x = 0$, then in terms of x, $\frac{dy}{dx} =$

A. $\frac{1}{x} + \sin x$
B. $\frac{1}{x} - \sin x$
C. $e^{-\cos x}$
D. $e^{-\cos x}\sin x$
E. $e^{\sin x}$

21. The coefficient of x^4 in the Taylor series expansion about $x = 0$ for $f(x) = e^{3x^2}$ is

A. $\frac{1}{24}$
B. $\frac{9}{4}$
C. $\frac{9}{2}$
D. $\frac{9}{24}$
E. $\frac{81}{24}$

22. If f is continuous on the closed interval $[a,b]$, then there exists c such that $a < c < b$ and $f'(c) =$

A. $f'(a)$
B. $f'(b)$
C. 0
D. $\frac{f'(b) - f'(a)}{b - a}$
E. $\frac{f(b) - f(a)}{b - a}$

23. If $f(x)=\sum_{n=1}^{\infty}\left(1-\frac{1}{x}\right)^n$, then $f(2)=$

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

24. $\lim_{x\to 0}\frac{e^x-1}{\sin x}=$

A. -1

B. 0

C. 1

D. e

E. nonexistent

25. If $y=\arcsin\left(e^{x^2}\right)$, then $\frac{dy}{dx}=$

A. $\frac{1}{\sqrt{1-e^{2x^2}}}$

B. $\frac{2x}{\sqrt{1-e^{2x^2}}}$

C. $\frac{2xe^{x^2}}{\sqrt{1-e^{2x^2}}}$

D. $\frac{2xe^{x^2}}{\sqrt{1-e^{x^2}}}$

E. $\frac{2xe^{x^2}}{\sqrt{1+e^{x^2}}}$

26. $\int x^2\sec x\tan x\,dx=$

A. $x^2\sec x-2\sec x$

B. $x^2\sec x-2x\sec x$

C. $x^2\sec x-2\int\sec x\,dx$

D. $x^2\sec x-\int x\sec x\,dx$

E. $x^2\sec x-2\int x\sec x\,dx$

27. Which of the following is not equal to $\int_0^{\pi/2}\cos x\,dx$?

A. $\int_0^{\pi/2}\sin x\,dx$

B. $\int_{\pi/2}^{\pi}\sin x\,dx$

C. $\int_{\pi/2}^{\pi}\cos x\,dx$

D. $\int_0^{3\pi/2}\sin 2x\,dx$

E. $\frac{1}{2}\int_{-\pi/3}^{\pi/3}\cos\frac{x}{2}\,dx$

28. If $\frac{dy}{dx}=\cos x$, then the average rate of change of y with respect to x on the closed interval $[0,\pi]$ is

A. 0

B. $-\frac{1}{\pi}$

C. $\frac{1}{\pi}$

D. π

E. $-\pi$

SECTION I, Part B
Time—50 Minutes
Number of Questions—17
A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems. After examining the choices, select the best answer. No credit will be given for anything written in the test book.

In this test:

1. The exact numerical value of the correct answer does not always appear among the answer choices given. When this happens, select the answer that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which *f(x)* is a real number.

29. Let $f(x) = \int_1^{2x-x^2} e^{1-t}\,dt$. At what value of x does $f(x)$ have a relative minimum?

A. $x = -1$
B. $x = 0$
C. $x = 1$
D. $x = 2$
E. No value of x

30. The length of the path described by the parametric equations $x = t + 1$ and $y = 3t - 1$, when $0 \le t \le 2$, is

A. 3.771
B. 3.986
C. 4.799
D. 6.070
E. 7.426

31. The length of the curve determined by the equations $x = t - 1$ and $y = \sqrt{t}$ from $t = 0$ to $t = 4$ is

A. 7.555
B. 8.161
C. 10.387
D. 10.736
E. 13.140

32. What is the volume of the solid generated by rotating about the x-axis the region in the first quadrant enclosed by the curve $y = 1 - \sin x$ and the x- and y-axes?

A. 0.342
B. 0.712
C. 1.119
D. 1.571
E. 4.712

33. The x-coordinate of the point on the curve $2x^2 - y = 1$ closest to the point $(-2, 0)$ is

A. –0.824
B. –0.707
C. –0.25
D. 0
E. 0.354

34. The area of the region inside the polar curve $r = 2 + 2\sin\theta$ and outside the polar curve $r = 2$ is given by

A. 3.142
B. 5.571
C. 6.283
D. 11.142
E. 22.283

35. $\lim_{x\to 2} \frac{\int_2^x e^{-t^2}\,dt}{x-2}$ is

A. $\frac{1}{e^4}$

B. $\frac{1}{e^2}$

C. $-\frac{e^4}{4}$

D. $-\frac{1}{4e^4}$

E. nonexistent

36. What is the approximation of the value of e^2 using the fourth-degree Taylor polynomial about $x = 0$ for e^x?

A. $\frac{1}{3}$

B. $\frac{4}{3}$

C. 7

D. 11

E. 31

37. $\int_{-1}^{1} \frac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}}dx =$

A. 0

B. 2

C. π

D. $2+\pi$

E. $2+\frac{\pi}{2}$

38. The region bounded by the x-axis and the graph of $y = \sin x$ is divided by the vertical line $x = k$. If the area of the region $0 \le x \le k$ is three times the area of the region $k \le x \le \pi$, $k =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $2+\pi$

E. $\frac{5\pi}{6}$

39. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $(\sin t, \cos 2t)$, then at time $t = 1$, its acceleration vector is

A. $(0.540, 0.909)$

B. $(0.540, -1.819)$

C. $(-0.841, 0.832)$

D. $(-0.841, 1.665)$

E. $(0.8414, -0.416)$

40. If $f(x) = \begin{cases} (x-2)^2 & \text{for } x \le 3 \\ \frac{1}{x-2} & \text{for } x > 3 \end{cases}$ then $\int_2^5 f(x)dx =$

A. 9

B. 8.667

C. 1.432

D. 1.099

E. 0.333

41. The slope of the line normal to the curve $x^2y - xy^2 = 2x$ at $(3,1)$ is

A. –0.6

B. –1

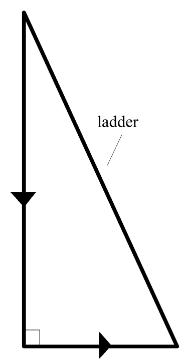
C. 1

D. 3

E. 4.5

42. Consider the curve in the xy-plane represented by $x = t^2 - 3t + 1$ and $y = t^2 - 3$ for $t \geq 0$. The slope of the tangent to the curve at $x = 5$ is

A. 0.7
B. 1.6
C. 5
D. 7
E. 8



43. A 20-foot ladder rests against a vertical wall, as shown in the figure above. If the ladder is sliding down the wall, how far is the foot of the ladder from the base of the wall at the moment when the top of the ladder is sliding down twice as fast as the foot of the ladder is moving away?

A. 4.472
B. 6.667
C. 8.944
D. 11.547
E. 14.142

44. What is the radius of the cone with maximum volume if the sum of the circumference and the height is 12 inches?

A. 0.785
B. 1.273
C. 2.546
D. 4
E. 8

45. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^{n+1}}$ is

A. $0 < x < 4$
B. $0 \leq x \leq 4$
C. $0 < x \leq 4$
D. $0 \leq x < 4$
E. All real x

SECTION II

GENERAL INSTRUCTIONS

You may wish to look over the problems before you begin, since you may not complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- You may use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps.
- Unless otherwise specified, answers (numeric or algebraic) do not need to be simplified. If your answer is given as a decimal approximation, it should correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A

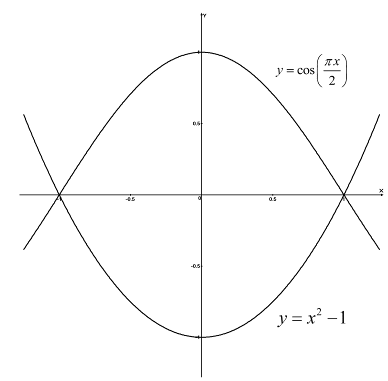
Time—30 minutes

Number of problems—2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you may use your calculator to solve an equation, find the derivative of a function, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem. If you use other built-in features, you must show the steps necessary to produce your results.



1. Let R be the region bounded by the graphs of $y = \cos\left(\frac{\pi x}{2}\right)$ and $y = x^2 - 1$, as shown in the figure above.

a. Find the area of R.

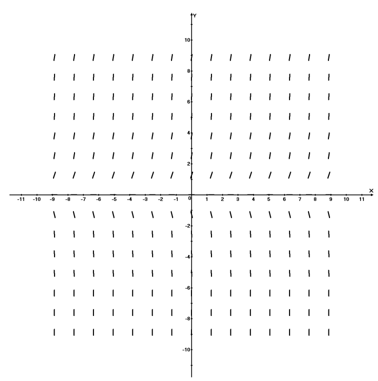
b. The line $y = k$ splits the region R into two equal parts. Find the value of k.

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle. Find the volume of this solid.

d. The region R models a lake. The depth of the lake at any point (x,y) is described by the function $h(x) = 10 - x^2$. Write, but do not evaluate, an expression involving an integral that can be used to find the volume of water in the lake.

2. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{4}(12 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 6$.

a. A slope field for this differential equation is given below. Sketch possible solution curves through the points $(0,6)$ and $(-3,-6)$.



b. Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

c. Write the third-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

d. Find the particular solution $y = f(t)$ with $f(0) = 6$.

PART B
Time—60 minutes
Number of problems—4
NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.
During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

Time	9 a.m.	10:30 a.m.	12 noon	1:30 p.m.	3 p.m.	4:30 p.m.	6 p.m.	7:30 p.m.	9 p.m.
t	0	1.5	3	4.5	6	7.5	9	10.5	12
$P(t)$	200	728	1193	1329	1583	1291	804	256	0

3. A free health care clinic was open to the public for 12 hours, beginning at 9 a.m. $(t = 0)$ and ending at 9 p.m. The number of people signing in to be seen for care at time $0 \le t \le 12$ is modeled by a twice-differentiable function P. Values of $P(t)$ at various times t are shown in the table above.

a. Use the data in the table to estimate the rate at which the number of people seeking care was changing at 5 p.m. $(t = 8)$. Show the computation that leads to your answer and indicate units of measure.

b. Use a midpoint Riemann sum with four intervals of equal size to estimate the total number of people seeking care during the 12-hour period.

c. If planners estimated that the rate, in people per hour, at which people could receive care was modeled by the function $r(t) = 400\pi \sin\left(\frac{\pi(t+3)}{15}\right)$ for $0 \le t \le 12$, how many people could receive care in the period from 11 a.m. $(t = 2)$ to 9 p.m. $(t = 12)$?

d. At what time during the period $0 \le t \le 12$ was the rate at which people were seen for care greatest? Estimate the number of people per hour that could be seen for care at the maximum rate. Was it higher or lower than the number of people signing in for care at that hour?

4. An object moving along a curve in the *xy*-plane is at position $(x(t), y(t))$ at time *t*, where $\frac{dx}{dt} = 6\cos\left(\frac{\pi t}{4}\right)$ and $\frac{dy}{dt} = 12\sin\left(\frac{\pi t}{4}\right)$ for $t \geq 0$. At time $t = 1$, object is at position $(1, -1)$.

a. Write an equation for the line tangent to the curve at time $t = 1$.

b. Find the acceleration vector and the speed of the object at time $t = 1$.

c. Write an integral expression that can be used to find the total distance traveled by the object over the time interval $0 \leq t \leq 2$.

d. Is there a time $t > 1$ at which the object is on the *x*-axis? Explain your reasoning.

5. The twice-differentiable function *f* is defined for all real numbers and satisfies the conditions $f(1) = 1$, $f'(1) = -1$, and $f''(1) = 0$.

a. The function *g* is given by $g(x) = f(x)\ln(x)$. Find $g'(1)$, and $g''(1)$. Show the work that leads to your answers.

b. The function *h* is given by $h(x) = f(x)\sin(\pi x)$. Find $h'(x)$ and write an equation for the line tangent to the graph of *h* at $x = 1$.

6. The derivative of a function f is given by $f'(x) = (2x - 1)e^{2x-1}$ and $f(1) = 3$.

a. The function has a single critical point. Find this point. Is this a relative maximum, a relative minimum, or neither?

b. On what intervals, if any, is the function decreasing and concave up? Explain your reasoning.

c. Find the value of $f(5)$.

SOLUTIONS: AP Calculus BC Practice Test

Multiple Choice

Section 1 Part A

1. C. The graph of $y = f(x)$ has two relative minima at $x = b$ and $x = f$ and one relative maximum at $x = d$. The graph of $y = f'(x)$ should have zeros at the corresponding *x*-values, which eliminates B, D, and E. $y = f'(x)$ should change from negative to positive at each of the relative minima, and from positive to negative at the relative maximum, which eliminates A.

2. E. If f has a relative minimum at *c*, then $f'(c) = 0$ and $f''(c) > 0$. None of the statements are true.

3. E. $\frac{dy}{dx} = x^2y$ would have all non-negative slopes above the *x*-axis and that is not the case. $\frac{dy}{dx} = xy^2$ would have all non-negative slopes to the right of the *y*-axis, also not the case. $\frac{dy}{dx} = xy + xy^2 = x(y + y^2)$ should have negative slopes throughout the second quadrant, since *x* is negative and $y + y^2$ is positive. $\frac{dy}{dx} = x^2y + xy = y(x^2 + x)$ should be negative in the fourth quadrant. Neither of these is true. $\frac{dy}{dx} = x^2y + xy^2$ is the only remaining possibility.

4. D. Use partial fraction decomposition. $\int \frac{dx}{x^2 - 7x - 18} = \int \frac{dx}{(x-9)(x+2)} = \int \frac{Adx}{x-9} + \int \frac{Bdx}{x+2}$. Since $Ax + 2A + Bx - 9B = 1$, $A = -B$ and $2A - 9B = 1$. Solve to find $A = \frac{1}{11}$ and $B = -\frac{1}{11}$, and then integrate. $\frac{1}{11}\int \frac{dx}{x-9} - \frac{1}{11}\int \frac{dx}{x+2} = \frac{1}{11}[\ln(x-9) - \ln(x+2)]$.

5. A. Use integration by parts with $u = x^2$, $du = 2xdx$, $dv = \sin x dx$, and $v = -\cos x$. $\int x^2 \sin x dx = -x^2 \cos x + \int 2x \cos x dx$. This requires that you use parts again, this time with $u = 2x$, $du = 2dx$, $dv = \cos x dx$, and $v = \sin x$. Then $\int x^2 \sin x dx = -x^2 \cos x + 2x \cos x dx = -x^2 \cos x + 2x \sin x - 2\int \sin x dx$ $= -x^2 \cos x + 2x \sin x + 2\cos x + C$.

6. B. If the position vector is $\langle t^2 - t^3, (2-3t)^2 \rangle$, the velocity vector is $\langle 2t - 3t^2, -6(2-3t) \rangle$, and the acceleration vector is $\langle 2-6t, 18 \rangle$. At $t = 2$, the acceleration vector is $\langle -10, 18 \rangle$.

7. B. If $\frac{dy}{dx} = \cos x \sin^2 x$, make a substitution with $u = \sin x$ and $du = \cos x$, and integrate to get $y = \frac{\sin^3 x}{3} + C$. If $y = 0$ when $x = 0$, $C = 0$ and $y = \frac{\sin^3 x}{3}$. Substitute $x = \frac{\pi}{2}$ to get $y = \frac{1}{3}\sin^3 \frac{\pi}{2} = \frac{1}{3}(1)^3 = \frac{1}{3}$.

8. D. If f is the function defined by $f(x) = 2x^6 - 9x^4$, then the first derivative $f'(x) = 12x^5 - 36x^3 = 0$ when $12x^3(x^2 - 3) = 0$. Solving gives you $x = 0$ and $x = \pm\sqrt{3}$. Check the sign of the first derivative for change of sign.

	$x < -\sqrt{3}$	$-\sqrt{3}$	$-\sqrt{3} < x < 0$	0	$0 < x < \sqrt{3}$	$\sqrt{3}$	$x > \sqrt{3}$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

9. E. If f is twice differentiable and if $g(x) = f(f(x))$, then $g'(x) = f'(f(x))f'(x)$ and $g''(x) = f'(f(x))f''(x) + f'(x)f''(f(x))f'(x) =$ $f'(f(x))f''(x) + [f'(x)]^2 f''(f(x))$.

10. C. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \ldots = \sum_{n=1}^{\infty} n a_n x^{n-1}$ and $f''(x) = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 + \ldots = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$. $f''(2) = \sum_{n=2}^{\infty} n(n-1)a_n 2^{n-2}$.

11. C. If $\lim_{k\to\infty}\int_1^k \frac{dx}{x^p}$ is finite, then $\sum_{n=1}^{\infty}\frac{1}{x^p}$ converges, which implies that $p > 1$. $-p < -1$ so $\sum_{n=1}^{\infty}\frac{1}{x^{-p}}$ diverges. $p^2 > 1$ and $|p| > 1$ so the two remaining series converge.

12. A. Solve $x = t + 3$ to get $t = x - 3$ and substitute into $y = (t-2)^2$ to get $y = (x-5)^2 = x^2 - 10x + 25$. The parabola has a y-intercept of 25.

13. E. $\int_0^{\infty} x^3 e^{2x^4} dx = \lim_{k\to\infty}\int_0^k x^3 e^{2x^4} dx$. If $u = 2x^4$ and $du = 8x^3 dx$, $\lim_{k\to\infty}\int_0^k x^3 e^{2x^4} dx$ $= \lim_{k\to\infty}\left[\frac{1}{8}e^{2x^4}\right]\Big|_0^k - \lim_{k\to\infty}\left[\frac{1}{8}e^{2k^4} - \frac{1}{8}\right] = \frac{1}{8}\lim_{k\to\infty}\left[e^{2k^4} - 1\right]$ which goes to positive infinity. The improper integral diverges.

14. D. $\frac{dP}{dt} = P\left(500 - \frac{P}{4}\right) = 500P\left(1 - \frac{P}{2000}\right)$ integrates to $P = \frac{2000}{1 + Ce^{-500t}}$. (If you don't recall this, separate the variables and integrate using a partial fraction decomposition.) Use the initial condition to find that $100 = \frac{2000}{1+C}$, so $100 + 100C = 2000$, $100C = 1900$ and $C = 19$. The equation for the population at time t is $P(t) = \frac{2000}{1 + 19e^{-500t}}$ and $\lim_{t\to\infty} P(t) = 2000$.

15. C. The function is decreasing when the first derivative is negative. If $f(x) = 3x^3 - 4x$, then $f'(x) = 9x^2 - 4$. First find the zeros of the derivative. If $9x^2 - 4 = 0$, $x = \pm\frac{2}{3}$. Check the sign of the derivative to determine where it is negative. $f'(-1) = 9 - 4 > 0$, $f'(0) = -4 < 0$ and $f'(1) = 9 - 4 > 0$, so the function is decreasing on the interval $-\frac{2}{3} < x < \frac{2}{3}$.

16. D. Given that $\frac{d}{dt}\left(\frac{1}{x}\right) = k\frac{dx}{dt}$, differentiate to get $-x^2\frac{dx}{dt} = k\frac{dx}{dt}$ and substitute $x = 3$, to get $-\frac{1}{9}\frac{dx}{dt} = k\frac{dx}{dt}$. Solve to find $k = -\frac{1}{9}$.

17. B. Check each series. For $\sum_{n=1}^{\infty}\frac{n+1}{n+3}$, $\lim_{n\to\infty}\frac{n+1}{n+3}=\lim_{n\to\infty}\left(1-\frac{2}{n+3}\right)=1$ and $\left|\frac{n+2}{n+4}\cdot\frac{n+3}{n+1}\right|=\left|\frac{n^2+5n+6}{n^2+5n+4}\right|>1$, so the series diverges. For $\sum_{n=1}^{\infty}\frac{\sin(n\pi)}{n}$, $\sin(n\pi)=0$ for all n, so the series converges. The final series, $\sum_{n=1}^{\infty}\frac{2}{n+1}=2\sum_{n=1}^{\infty}\frac{1}{n+1}=2\sum_{n=2}^{\infty}\frac{1}{n}$ is a multiple of a harmonic series. The harmonic series diverges, therefore, $\sum_{n=1}^{\infty}\frac{2}{n+1}$ diverges.

18. A. $y=\frac{x^2-k}{x-1}$ will have a relative minimum at $x=3$ if the derivative $y'=\frac{(x-1)(2x)-(x^2-k)(1)}{(x-1)^2}=\frac{x^2-2x+k}{(x-1)^2}$ is equal to zero at $x=3$, and changes from negative to positive. Substitute $x=3$ into the derivative and solve $3^2-2(3)+k=0$ to get $k=-3$. Check for the proper change of sign, being careful to avoid the discontinuity at $x=1$. If $k=-3$, $y'=\frac{x^2-2x-3}{(x-1)^2}$ so $y'(2)=\frac{4-4-3}{1^2}=-3$ and $y'(4)=\frac{16-8-3}{3^2}=\frac{5}{9}$.

19. C. If $h(x)=f(x)g(x)(1+g(x))=f(x)g(x)+f(x)[g(x)]^2$, then $h'(x)=f(x)g'(x)+g(x)f'(x)+f(x)\cdot 2g(x)g'(x)+f'(x)[g(x)]^2$ and when $x=1$, $h'(1)=f'(1)[g(1)+[g(1)]^2]+g'(1)[f(1)+2f(1)g(1)]$. Substituting $g(1)=0$, $h'(1)=f'(1)[0+0]+g'(1)[f(1)+2f(1)\cdot 0]=f(1)g'(1)$.

20. D. If $\ln y+\cos x=0$, then $y=e^{-\cos x}$ and $\frac{dy}{dx}=e^{-\cos x}\sin x$.

21. C. $f(x)=e^{3x^2}$ Start with $e^x=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\ldots+\frac{x^n}{n!}+\ldots$

$e^{3x^2}=1+3x^2+\frac{(3x^2)^2}{2}+\frac{(3x^2)^3}{6}+\ldots\frac{(3x^2)^n}{n!}+\ldots$

$=1+3x^2+\frac{9x^4}{2}+\frac{27x^6}{6}+\ldots+\frac{3^nx^{2n}}{n!}+\ldots$ The coefficient of x^4 is $\frac{9}{2}$.

22. E. If f is continuous on the closed interval $[a,b]$, then the Mean Value Theorem guarantees that there exists c such that $a<c<b$ and $f'(c)=\frac{f(b)-f(a)}{b-a}$. While the other values might be possible, none is certain.

23. C. If $f(x)=\sum_{n=1}^{\infty}\left(1-\frac{1}{x}\right)^n$, then $f(2)=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n=1$.

24. C. $\lim_{x\to 0}\frac{e^x-1}{\sin x}=\lim_{x\to 0}\frac{e^x}{\cos x}=1$.

25. C. If $y=\arcsin(e^{x^2})$, then $\frac{dy}{dx}=\frac{1}{\sqrt{1-(e^{x^2})^2}}\cdot(2xe^{x^2})=\frac{2xe^{x^2}}{\sqrt{1-e^{2x^2}}}$.

26. E. Use integration by parts, with $u=x^2$, $du=2xdx$, $dv=\sec x\tan xdx$, and $v=\sec x$. Then $\int x^2\sec x\tan xdx=x^2\sec x-2\int x\sec xdx$.

27. C. The first few choices can be estimated geometrically. $\int_0^{\pi/2}\sin xdx$ and $\int_{\pi/2}^{\pi}\sin xdx$ will be equal to each other, and to the given integral, but $\int_{\pi/2}^{\pi}\cos xdx$ will be the negative of the given integral. Integrate to check the remaining options. $\int_0^{3\pi/2}\sin 2xdx=-\frac{1}{2}\cos 2x\Big|_0^{\frac{3\pi}{2}}=1$ and $\frac{1}{2}\int_{-\pi/3}^{\pi/3}\cos\frac{x}{2}dx=\frac{1}{2}\cdot 2\sin\frac{x}{2}\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}=1$.

28. A. If $\frac{dy}{dx}=\cos x$, then the average rate of change of y with respect to x on the closed interval $[0,\pi]$ is $\frac{1}{\pi-0}\int_0^{\pi}\cos xdx=\frac{1}{\pi}[-\sin x]_0^{\pi}=\frac{1}{\pi}(0+0)=0$.

Section I, Part B

29. E. Find $f'(x) = e^{1-2x+x^2}(2-2x)$, and consider when the derivative is equal to zero. $e^{1-2x+x^2}(2-2x) = 0$ only at $x = 1$, since $e^{1-2x+x^2} > 0$ for all x. Check to see if this critical number is in fact a minimum. When $x < 1$, the derivative is positive, and negative when $x > 1$, so the critical point is a relative maximum, not a minimum. Therefore the function has no relative minimum.

30. D. The length of the path described by the parametric equations $x = t + 1$ and $y = 3t - 1$, when $0 \le t \le 2$, is given by $\int_0^2 \sqrt{(t+1)^2 + (3t-1)^2}\,dt$ $= \int_0^2 \sqrt{t^2 + 2t + 1 + 9t^2 - 6t + 1}\,dt = \int_0^2 \sqrt{10t^2 - 4t + 2}\,dt \approx 6.070$.

31. A. The length of the curve determined by the equations $x = t - 1$ and $y = \sqrt{t}$ from $t = 0$ to $t = 4$ is $\int_0^4 \sqrt{(t-1)^2 + \sqrt{t}^{\,2}}\,dt = \int_0^4 \sqrt{t^2 - t + 1}\,dt \approx 7.555$.

32. C. The volume of the solid generated by rotating about the x-axis the region in the first quadrant enclosed by the curve $y = 1 - \sin x$ and the x- and y-axes is $\pi \int_0^{\pi/2} (1 - \sin x)^2\,dx \approx 0.356\pi \approx 1.119$.

33. A. Each point on the curve $2x^2 - y = 1$ is of the form $(x, 2x^2 - 1)$. Consider the distance from $(x, 2x^2 - 1)$ to $(-2, 0)$. $d = \sqrt{(x+2)^2 + (2x^2-1)^2}$ $= \sqrt{x^2 + 4x + 4 + 4x^4 - 4x^2 + 1} = \sqrt{4x^4 - 3x^2 + 4x + 5}$. To find the minimum distance, take the derivative. $d' = \dfrac{16x^3 - 6x + 4}{2\sqrt{4x^4 - 3x^2 + 4x + 5}} = \dfrac{8x^3 - 3x + 2}{\sqrt{4x^4 - 3x^2 + 4x + 5}}$. Set the derivative equal to zero and solve by calculator. $x \approx -0.824$. Check the change of sign of the derivative to verify this point is a minimum. $d'(-1) = \dfrac{8x^3 - 3x + 2}{\sqrt{4x^4 - 3x^2 + 4x + 5}} = -\dfrac{3}{2} < 0$ and $d'(0) = \dfrac{8x^3 - 3x + 2}{\sqrt{4x^4 - 3x^2 + 4x + 5}} = \dfrac{2}{\sqrt{5}} > 0$. The x-coordinate of the nearest point is -0.824.

34. D. Find the points of intersection by solving $2 + 2\sin\theta = 2$ and $\theta = 0$ or $\theta = \pi$. The area of the region is $A = \frac{1}{2}\int_0^\pi [(2 + 2\sin\theta)^2 - 2^2]\,d\theta =$ $\frac{1}{2}\int_0^\pi [8\sin\theta + 4\sin^2\theta]\,d\theta = \int_0^\pi [4\sin\theta + 2\sin^2\theta]\,d\theta = \int_0^\pi [4\sin\theta + 1 - \cos 2\theta]\,d\theta$ $= -4\cos\theta + \theta - \frac{1}{2}\sin 2\theta\,\Big|_0^\pi = (4 + \pi) - (-4) = 8 + \pi \approx 11.142$.

35. A. $\lim\limits_{x\to 2} \dfrac{\int_2^x e^{-t^2}\,dt}{x - 2} = \lim\limits_{x\to 2} \dfrac{e^{-x^2}}{1} = \dfrac{1}{e^4}$

36. C. The approximate value of e^2 using the fourth-degree Taylor polynomial about $x = 0$ for e^x is $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$ evaluated at $x = 2$, or $e^2 \approx 1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} \approx 7$.

37. D. $\int_{-1}^{1} \dfrac{1 + \sqrt{1 - x^2}}{\sqrt{1 - x^2}}\,dx = \int_{-1}^{1}\left(1 + \dfrac{1}{\sqrt{1 - x^2}}\right)dx = x + \sin^{-1}x\,\Big|_{-1}^{1} =$ $\left(1 + \frac{\pi}{2}\right) - \left(-1 - \frac{\pi}{2}\right) = 2 + \pi$

38. E. $\int_0^k \sin x\,dx = 3\int_0^\pi \sin x\,dx$ means that $-\cos x\,\Big|_0^k = 3[-\cos x\,\Big|_k^\pi]$. Evaluate and solve $-\cos k + 1 = 3[1 + \cos k]$ to find that $\cos k = -\frac{1}{2}$ so $k = \frac{5\pi}{6}$.

39. D. If the position vector is $(\sin t, \cos 2t)$, then the velocity vector is $(\cos t, -2\sin 2t)$, and the acceleration vector is $(-\sin t, -4\cos 2t)$. At time $t = 1$, its acceleration vector is $(-\sin 1, -4\cos 2) = (-0.8414, 1.665)$.

40. C. $\int_2^5 f(x)\,dx = \int_2^3 (x-2)^2\,dx + \int_3^5 \dfrac{dx}{x-2} = \int_2^3 (x^2 - 4x + 4)\,dx + \int_3^5 \dfrac{dx}{x-2}$ $= \frac{x^3}{3} - 2x^2 + 4x\,\Big|_2^3 + \ln(x-2)\,\Big|_3^5 = (9 - 18 + 12) - \left(\frac{8}{3} - 8 + 8\right) + \ln 3 - \ln 1 =$ $3 - \frac{8}{3} + \ln 3 = \frac{1}{3} + \ln 3 \approx 1.432$.

41. C. To find the slope of the line normal to the curve $x^2y - xy^2 = 2x$, first differentiate implicitly. $x^2\frac{dy}{dx} + 2xy - 2xy\frac{dy}{dx} - y^2 = 2$, so $x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = 2 - 2xy + y^2$ and $\frac{dy}{dx} = \frac{2 - 2xy + y^2}{x^2 - 2xy}$. At $(3,1)$, $\frac{dy}{dx} = \frac{2 - 2(3)(1) + 1}{9 - 2(3)(1)} = \frac{2 - 6 + 1}{9 - 6} = \frac{-3}{3} = -1$ so slope of the tangent is -1 and the slope of the normal is the negative reciprocal, or 1.

42. B. Find $\frac{dx}{dt} = 2t - 3$ and $\frac{dy}{dt} = 2t$. Then $\frac{dy}{dx} = \frac{2t}{2t - 3}$. When $x = 5$, $5 = t^2 - 3t + 1$. Solve $t^2 - 3t - 4 = 0$ to find $t = 4$ or $t = -1$, but reject $t = -1$ since $t \geq 0$. $\frac{dy}{dx} = \frac{2t}{2t - 3} = \frac{8}{5} = 1.6$

43. C. The basic relationship is the Pythagorean Theorem with $c = 20$ feet so $a^2 + b^2 = 400$. Differentiate to get $2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$, and substitute, using $\frac{da}{dt} = 2\frac{db}{dt}$. $a\left(2\frac{db}{dt}\right) + b\frac{db}{dt} = 0$ means that $\frac{db}{dt}(2a + b) = 0$, and since we know $\frac{db}{dt}$ is not zero, because the ladder is moving, it must be true that $2a + b = 0$ or $b = -2a$. Substitute into $a^2 + b^2 = 400$ to get $a^2 + 4a^2 = 400$ and solve to find $a = 4\sqrt{5} \approx 8.944$.

44. B. If $C + h = 2\pi r + h = 12$ then $h = 12 - 2\pi r$ and $V = \frac{1}{3}\pi r^2 h$ becomes $V = \frac{1}{3}\pi r^2(12 - 2\pi r)$ or $V = 4\pi r^2 - \frac{2}{3}\pi^2 r^3$. Differentiate and set the derivative equal to zero. $V' = 8\pi r - 2\pi^2 r^2 = 0$ factors to $2\pi r(4 - \pi r) = 0$ so $r = 0$ or $r = \frac{4}{\pi} \approx 1.273$.

45. A. Find the radius of convergence. $\left|\frac{(x-2)^{n+1}}{2^{n+2}} \cdot \frac{2^{n+1}}{(x-2)^n}\right| = \left|\frac{x-2}{2}\right|$ and $\left|\frac{x-2}{2}\right| < 1$ when $-1 < \frac{x-2}{2} < 1$ or $0 < x < 4$. Then check the endpoints of the interval. When $x = 0$, $\sum_{n=0}^{\infty}\frac{(-2)^n}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{2}$. Since $\left|\frac{(-1)^{n+1}}{2} \cdot \frac{2}{(-1)^n}\right| = 1$, the series does not converge. When $x = 4$, $\sum_{n=0}^{\infty}\frac{(4-2)^n}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{2^n}{2^{n+1}} = \sum_{n=0}^{\infty}\frac{1}{2}$, which also diverges. The interval of convergence is the open interval $(0,4)$.

Section II, Part A

1.

a. $A = \int_{-1}^{1}\left(\cos\left(\frac{\pi x}{2}\right) - (x^2 - 1)\right)dx = \int_{-1}^{1}\cos\left(\frac{\pi x}{2}\right)dx - \int_{-1}^{1}(x^2 - 1)dx =$ $\frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big|_{-1}^{1} - \left(\frac{x^3}{3} - x\right)\Big|_{-1}^{1} = \frac{2}{\pi}(1) - \frac{2}{\pi}(-1) - \left(-\frac{2}{3} - \frac{2}{3}\right) = \frac{4}{\pi} + \frac{4}{3} \approx 2.607$.

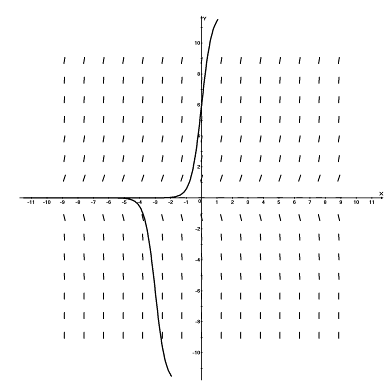
b. From a., you know that the total area is $= \frac{4}{\pi} + \frac{4}{3} \approx 2.607$ and you can deduce that $\int_{-1}^{1}\left(\cos\left(\frac{\pi x}{2}\right)\right)dx = \frac{4}{\pi}$ and $\int_{-1}^{1}(x^2 - 1)dx = -\frac{4}{3}$. Since $\frac{4}{\pi} < \left|-\frac{4}{3}\right|$, you can expect that $k < 0$. Therefore, the area of the region below $y = k$ is $\int_{-1}^{1}(k - x + 1)dx = \left(kx - \frac{x^3}{3} + x\right)\Big|_{-1}^{1} = \left(k - \frac{1}{3} + 1\right) - \left(-k + \frac{1}{3} - 1\right) = 2k + \frac{4}{3}$. Then $2k + \frac{4}{3} = \frac{1}{2}(2.607)$ so $2k + \frac{4}{3} = 1.3035$ and $k = -0.015$.

c. The volume of the solid is $V = \int_{-1}^{1}\frac{\pi}{2}\left(\frac{\cos\left(\frac{\pi x}{2}\right) - x^2 + 1}{2}\right)^2 dx =$ $\frac{\pi}{8}\int_{-1}^{1}\left(\cos\left(\frac{\pi x}{2}\right) - x^2 + 1\right)^2 dx = \frac{\pi}{8}(4.131) = 1.622$.

d. The volume of water in the lake is $\int_{-1}^{1} R \cdot h(x)dx =$ $\int_{-1}^{1}\left(\cos\left(\frac{\pi x}{2}\right) - (x^2 - 1)\right)(10 - x^2)dx$.

2.

a. The requested curves pass through the given points, follow the direction of the field, and approach zero as $x \to -\infty$.



b. Using Euler's method, with the given initial point $(0,6)$ and intervals of width $\frac{1}{2}$, find $\frac{dy}{dt} = \frac{6}{4}(12-6) = 9$ and $y_1 = 6 + 9 \cdot \frac{1}{2} = 10.5$. Repeat for the second interval. $\frac{dy}{dt} = \frac{10.5}{4}(12-10.5) = \frac{15.75}{4} = 3.9375$ and $y_2 = 10.5 + 3.9375 \cdot \frac{1}{2} = 12.46875$. $f(1) \approx 12.469$.

c. Given $\frac{dy}{dt} = \frac{y}{4}(12-y)$, find $\frac{d^2y}{dx^2} = \frac{y}{4}\left(-1\frac{dy}{dt}\right) + (12-y)\frac{1}{4}\frac{dy}{dt}$. Substitute $\frac{dy}{dt} = \frac{y}{4}(12-y)$ and simplify to get $9y - \frac{9}{4}y^2 + \frac{y^3}{8}$. Find the third derivative $\left(9 - \frac{9}{2}y + \frac{3}{8}y^2\right)\frac{dy}{dt}$. Evaluate the derivatives at $x = 0$ and $y = 6$. $\frac{dy}{dt} = \frac{6}{4}(12-6) = 9$, $\frac{d^2y}{dx^2} = \frac{6^2}{16}(12-6) + \frac{6}{16}(12-6)^2 = 0$, and the third derivative is equal to $\left(9 - 27 + \frac{27}{2}\right)9 = -\frac{81}{2}$. The third-degree Taylor polynomial for f about $t = 0$ is $f(x) \approx 6 + 9x + \frac{0}{2!}x^2 + \frac{-\frac{81}{2}}{3!}x^3 = 6 + 9x - \frac{27}{4}x^3$ or $f(x) \approx 6 + 9x - \frac{27}{4}x^3$, so $f(1) \approx 8.25$.

d. The logistic differential equation $\frac{dy}{dt} = \frac{y}{4}(12-y) = 3y\left(1 - \frac{y}{12}\right)$ integrates, with the help of partial fractions, to $y = \frac{12}{1+e^{-3t}}$. Recall that $\frac{dy}{y\left(1-\frac{y}{12}\right)} = 3dt$ becomes $\int\frac{dy}{y} - \frac{1}{12}\int\frac{dy}{1-\frac{y}{12}} = 3dt$, which integrates to $\ln y - \ln(12-y) = 3t + C$ or $y = \frac{12e^{3t+C}}{e^{3t+C}+1}$. Since $f(0) = 6$, $6 = \frac{12e^C}{e^C+1}$ and $C = 1$.

Section II, Part B

3.

a. The rate at which the number of people seeking care was changing at 5 p.m. was decreasing at a rate of $\frac{804-1291}{9-7.5} = -\frac{387}{1.5} = -258$ people per hour.

b. Use a midpoint Riemann sum with four 3-hour intervals. The total number of people seeking care during the 12-hour period was $3(728) + 3(1329) + 3(1291) + 3(256) = 2184 + 3987 + 3873 + 768 =$ 10813 people.

c. The number of people who could receive care was

$$\int_2^{12} 400\pi\sin\left(\frac{\pi(t+3)}{15}\right)dt = 400\pi\int_2^{12}\sin\left(\frac{\pi(t+3)}{15}\right)dt = -400\pi\cdot\frac{15}{\pi}\cos\left(\frac{\pi(t+3)}{15}\right)\Big|_2^{12} =$$

$-6000\cos\pi + 6000\cos\frac{\pi}{3} = 6000 + 6000\cdot\frac{1}{2} = 9000$ people.

d. $r(t) = 400\pi\sin\left(\frac{\pi(t+3)}{15}\right)$ is at a maximum when $r'(t) = 400\pi\cdot\frac{\pi}{15}\cos\left(\frac{\pi(t+3)}{15}\right) = 0$. Solve $\frac{80\pi^2}{3}\cos\left(\frac{\pi(t+3)}{15}\right) = 0$ or $\cos\left(\frac{\pi(t+3)}{15}\right) = 0$. If $\frac{\pi(t+3)}{15} = \frac{\pi}{2}$, $2t + 6 = 15$ and $t = 4.5$. $r(4.5) = 400\pi\sin\left(\frac{\pi}{2}\right) = 400\pi \approx 1256.637$ which is less than the 1329 people signing in for care at $t = 4.5$.

4.

a. Find $\frac{dx}{dt} = 6\cos\left(\frac{\pi t}{4}\right)$ and $\frac{dy}{dt} = 12\sin\left(\frac{\pi t}{4}\right)$, and divide to find $\frac{dy}{dx} = \frac{12\sin\left(\frac{\pi t}{4}\right)}{6\cos\left(\frac{\pi t}{4}\right)} = 2\tan\left(\frac{\pi t}{4}\right)$. When $t = 1$, $\frac{dy}{dt} = 2\tan\frac{\pi}{4} = 2$. The equation for the line tangent to the curve at time $t = 1$ is $y + 1 = 2(x-1)$ or $y = 2x - 3$.

b. If the velocity vector is $v(t) = \left\langle 6\cos\left(\frac{\pi t}{4}\right), 12\sin\left(\frac{\pi t}{4}\right)\right\rangle$, then the acceleration vector is $a(t) = \left\langle -\frac{3\pi}{2}\sin\left(\frac{\pi t}{4}\right), 3\pi\cos\left(\frac{\pi t}{4}\right)\right\rangle$. When $t = 1$, $v(1) = \left\langle 6\cos\left(\frac{\pi}{4}\right), 12\sin\left(\frac{\pi t}{4}\right)\right\rangle = \langle 3\sqrt{2}, 6\sqrt{2}\rangle$ and $a(1) = \left\langle -\frac{3\pi}{2}\cdot\frac{\sqrt{2}}{2}, 3\pi\cdot\frac{\sqrt{2}}{2}\right\rangle$ $= \left\langle \frac{3\sqrt{2}\pi}{4}, \frac{3\sqrt{2}\pi}{2}\right\rangle$. The speed of the object is $\sqrt{(3\sqrt{2})^2 + (6\sqrt{2})^2} =$ $\sqrt{18+72} = \sqrt{90} = 3\sqrt{10}$.

c. The total distance traveled by the object over the time interval $0 \le t \le 2$ is $\int_0^2 \sqrt{\left(6\cos\left(\frac{\pi t}{4}\right)\right)^2 + \left(12\sin\left(\frac{\pi}{4}t\right)\right)^2}\,dt = 6\int_0^2 \sqrt{\cos^2\left(\frac{\pi t}{4}\right) + 4\sin^2\left(\frac{\pi}{4}t\right)}\,dt =$ $6\int_0^2 \sqrt{1+3\sin^2\left(\frac{\pi t}{4}\right)}\,dt$.

d. If the object is on the x-axis, then $y(t) = 0$. $y(t) = y(1) + \int_1^t 12\sin\left(\frac{\pi t}{4}\right)dt$ $= -1 - \left[\frac{48}{\pi}\cos\left(\frac{\pi t}{4}\right)\right]_1^t = -1 - \left[\frac{48}{\pi}\cos\left(\frac{\pi t}{4}\right) - \frac{24\sqrt{2}}{\pi}\right] = -1 + \frac{24\sqrt{2}}{\pi} - \frac{48}{\pi}\cos\left(\frac{\pi t}{4}\right)$. Since $y(t)$ is continuous and $y(1) < 0$ but $y(2) = -1 + \frac{24\sqrt{2}}{\pi} > 0$, $y(t) = 0$ for some $1 < t < 2$.

5. $f'(1) = -1$, and $f''(1) = 0$

a. If $g(x) = f(x)\ln x$, $g'(x) = \frac{f(x)}{x} + f'(x)\ln x$. When $x = 1$, $g'(1) = f(1) = 1$. $g''(x) = \frac{xf'(x) - f(x)}{x^2} + \frac{f'(x)}{x} + f''(x)\ln x$. When $x = 1$, $g''(1) = 2f'(1) - f(1)$ $= -2 - 1 = -3$.

b. If $h(x) = f(x)\sin(\pi x)$, $h'(x) = \pi f(x)\cos(\pi x) + f'(x)\sin(\pi x)$. When $x = 1$, $h(1) = \sin(\pi) = 0$ and $h'(1) = \pi\cos(\pi) - \sin(\pi) = -\pi$. The equation for the line tangent to the graph of h at $x = 1$ is $y - 0 = -\pi(x-1)$ or $y = -\pi x + \pi$.

6. The derivative of a function f is given by $f'(x) = (2x-1)e^{2x-1}$ and $f(1) = 3$.

a. Since $f'(0) = -e^{2x-1} < 0$ and $f'(1) = e^{2x-1} > 0$, the critical point at $x = \frac{1}{2}$ is a relative minimum.

b. If the function is decreasing and concave up, the first derivative is negative and the second derivative is positive. $f'(x) = (2x-1)e^{2x-1} < 0$ when $x < \frac{1}{2}$. $f''(x) = 2(2x-1)e^{2x-1} + 2e^{2x-1} = 2e^{2x-1}(2x-1+1) = 4xe^{2x-1} > 0$ when $x > 0$. The function is decreasing and concave up when $0 < x < \frac{1}{2}$.

c. $f(5) = f(1) + \int_1^5 (2x-1)e^{2x-1}dx$ and this requires integration by parts with $u = 2x-1$, $du = 2dx$, $dv = e^{2x-1}dx$, and $v = \frac{1}{2}e^{2x-1}$. Then $f(5) = f(1) + \int_1^5 (2x-1)e^{2x-1}dx = f(1)\left[\frac{(2x-1)}{2}e^{2x-1}\right]_1^5 - \int_1^5 e^{2x-1}dx =$ $f(1) + \left[\frac{(2x-1)}{2}e^{2x-1}\right]_1^5 - \left[\frac{1}{2}e^{2x-1}\right]_1^5 = f(1) + \left[(x-1)e^{2x-1}\right]_1^5 = 3 + 4e^9$.