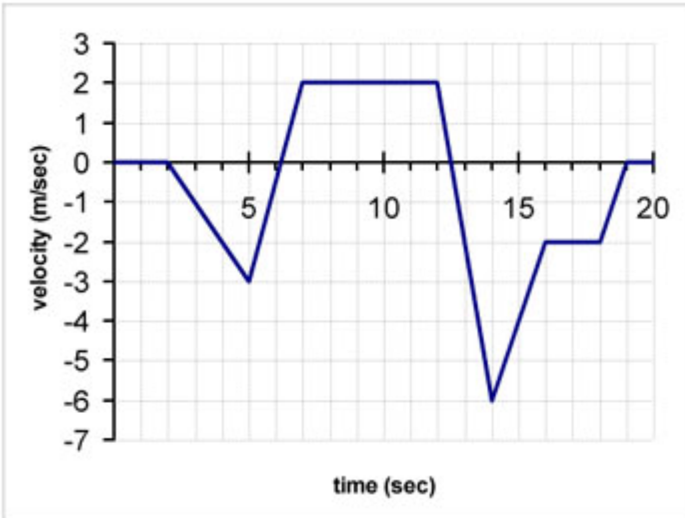
AP Review 11: Particle Motion (AB/BC Split)

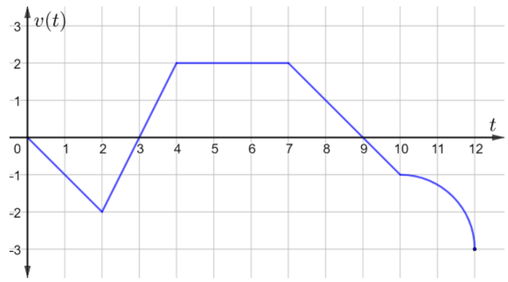
Particle Motion Reference Information	
For $a \le t \le b$, a particle moves along the x axis. Let $a \le c \le b$.	
$x(t) =$ position $\quad v(t) =$ velocity $\quad a(t) =$ acceleration $x'(t) = v(t)$ and $x''(t) = v'(t) = a(t)$	
Direction of Motion $v(t) > 0 \rightarrow$ moving right $v(t) < 0 \rightarrow$ moving left	**Speeding Up and Slowing Down** $v(t)$ and $a(t)$ same sign $\rightarrow$ speeding up $v(t)$ and $a(t)$ opposite signs $\rightarrow$ slowing down
Position at Time c $x(c) = x(a) + \int_a^c v(t)dt$	**Total Distance Traveled Over $[a, b]$** $\int_a^b \lvert v(t) \rvert dt$
Particle Changes Direction $v(t)$ changes sign	**Speed** $\lvert v(t) \rvert$



What can we know from looking at a graph of velocity? Let's label.

Connecting AB to BC: Fundamental Theorem of Calculus

Quick Check



Quick Check 1: For $0 \leq t \leq 12$ seconds, a particle moves along the x axis. The velocity, in meters/second, of the particle is modeled by the function $v(t)$, which is shown in the figure above and consists of four line segments and a quarter circle centered at $(10, -3)$. The position of the particle, $x(t)$, at time $t = 3$ is -2.

(**a**) For $0 \leq t \leq 12$ when is the particle moving to the left? Give a reason for your answer.

(**b**) Find the position of the particle at time $t = 12$ seconds.

(**c**) Find $\frac{1}{9}\int_0^9 v(t)dt$. Using correct units, interpret the meaning of the integral in context of the problem.

t (seconds)	0	1	4	6
$x(t)$	-1	3	-2	-4
$v(t)$ (feet per second)	0	5	-3	2
$a(t)$	6	-4	-1	1

Quick Check 2: A particle moves along the x axis with position at time t given by $x(t)$ for $0 \le t \le 6$ The velocity and acceleration of the particle are given by the differentiable functions $v(t)$ and $a(t)$ respectively. Selected values of $v(t)$, measured in feet per second, and $a(t)$ are given in the table above.

(**a**) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(**b**) Using correct units, explain the meaning of the definite integral $\int_0^6 |v(t)|dt$ in the context of the problem.

Approximate the value of $\int_0^6 |v(t)|dt$ using a right Riemann sum with the three subintervals indicated in the table.

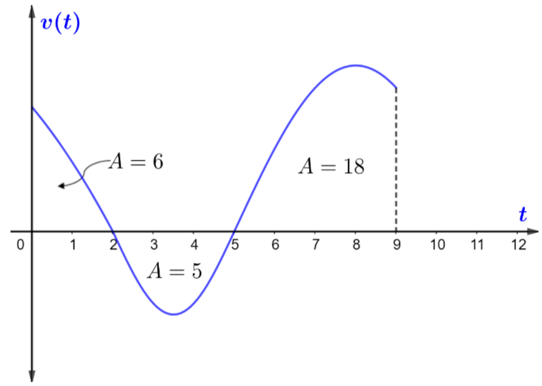
(**c**) At what time t is it certain that the particle is moving toward the origin? Explain your reasoning.

Quick Check 3: A particle is moving along the x axis on the interval $1 \leq t \leq 4$. Match the phrases with the appropriate expressions.

1. Displacement from $t = 1$ to $t = 4$	**A.** $\int_1^4 \lvert v(t) \rvert dt$
2. Position at time $t = 1$	**B.** $\frac{1}{3}\int_1^4 a(t)dt$
3. Total distance traveled from $t = 1$ to $t = 4$	**C.** $x(1) + \int_1^4 v(t)dt$
4. Position at time $t = 4$	**D.** $\frac{1}{3}\int_1^4 v(t)dt$
5. Average acceleration from $t = 1$ to $t = 4$	**E.** $\int_1^4 v(t)dt$
6. Average velocity from $t = 1$ to $t = 4$	**F.** $x(4) - \int_1^4 v(t)dt$

Free Response Practice: Connecting AB to BC

2020 FRQ Practice Problem BC1



BC1: For $t \geq 0$, the position of a particle moving along the x axis can be modeled by the twice differentiable function $x(t)$. A portion of the graph of v(t), the velocity of the particle, is shown in the figure above. The areas enclosed by the graphs of v and the t axis on the intervals $[0, 2]$, $[2, 5]$, and $[5,9]$ are 6, 5, and 18 respectively. For $t \geq 9$, the velocity of the particle can be modeled by the function $v_p(t) = 45e^{-kt}$ where $0 < k < 1$. It is known that $x(2) = 7$.

(**a**) Find all times t in the interval $0 \leq t \leq 9$ at which the particle changes direction. Give a reason for your answer.

(**b**) Write an expression involving an integral that gives the position $x(t)$. Use this expression to find the position of the particle at time $t = 0$.

(**c**) Find $\lim_{t \to \infty} x(t)$ in terms of k.

Homework

1. Add to the AP Calculus BC Review doc!
2. AB Test Takers, do the FRQ on this page and the assigned Khan Sets (4). For the FRQ, score yourself, make corrections, email me your work, and submit the score report as usual. BC Test Takers, complete the AB FRQ here and then work through the College Board practice FRQs. You will then check your work using the scoring guides, correct the work, and fill out the score report as usual. BUT the College Board practice FRQs are not "real." They are for learning and so doing have a scoring rubric. You will self-assess your learning on the score report.

AB 2012 #6 No Calc

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

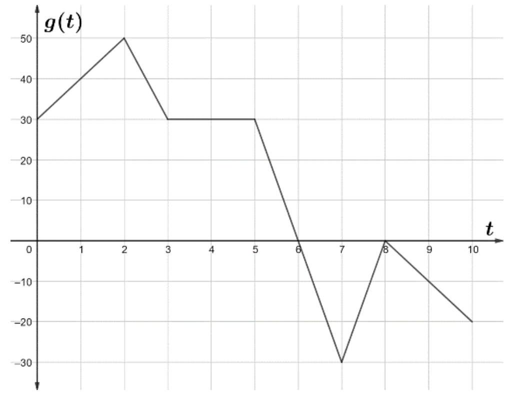
(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

2020 FRQ Practice Problem BC2

t	0	1	2	4	6	8	10	13
$f(t)$	7.5	21	27.5	32	45	57	70	100



BC2: The functions f and g are continuous for all $t \geq 0$. Selected values for the function f and a portion of the graph for g are shown above. For $t \geq 0$ the function f is increasing.

Part I: Particles P and Q are traveling along the x axis with positions $x_P(t)$ and $x_Q(t)$ respectively. The velocities of particles P and Q can be modeled by the functions $v_P(t) = f(t)$ and $v_Q(t) = g(t)$.
It is known that $x_P(1) = 5$.

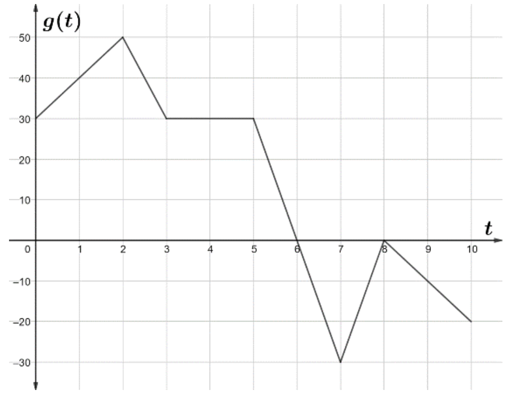
(**a**) Find the total distance traveled for particle Q from $t = 5$ to $t = 10$.

(**b**) Find all times on the open interval $(0, 10)$ when particle Q is speeding up. Give a reason for your answer.

(**c**) Approximate the acceleration of particle P at time $t = 7$. Show the computations that lead to your answer.

The problem has been restated.

t	0	1	2	4	6	8	10	13
$f(t)$	7.5	21	27.5	32	45	57	70	100



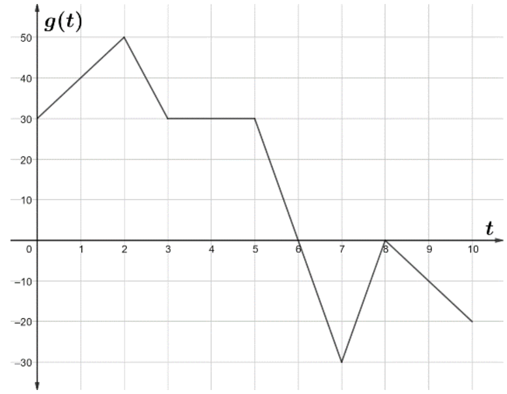
BC2: The functions f and g are continuous for all $t \geq 0$. Selected values for the function f and a portion of the graph for g are shown above. For $t \geq 0$ the function f is increasing.

Part I: Particles P and Q are traveling along the x axis with positions $x_P(t)$ and $x_Q(t)$ respectively. The velocities of particles P and Q can be modeled by the functions $v_P(t) = f(t)$ and $v_Q(t) = g(t)$.
It is known that $x_P(1) = 5$.

(**d**) $x_Q(6) = 105$. There is a value k such that $x_Q(k) = v_Q(k) = 0$ and for $10 \leq t \leq c$, the graph of $v_Q(t)$ is linear. Find c.

(**e**) The conditions for the Mean Value Theorem are met for $v_P(t)$ on the interval $[0, 2]$ and $t = 1$ satisfies the conclusion of the Mean Value Theorem for this interval. Find the second degree Taylor polynomial for $x_P(t)$ centered at t = 1 and use it to approximate $x_P(3)$.

The problem has been restated.



t	0	1	2	4	6	8	10	13
$f(t)$	7.5	21	27.5	32	45	57	70	100

BC2: The functions f and g are continuous for all $t \geq 0$. Selected values for the function f and a portion of the graph for g are shown above. For $t \geq 0$ the function f is increasing.

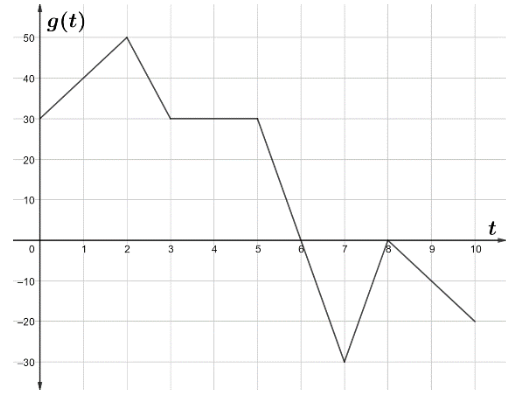
Part II: During the halftime intermission at a college football game, popcorn is made and sold at several concession stands throughout the stadium. For $0 \leq t \leq 10$, the rate that boxes of popcorn are made can be modeled by $M(t)$ and the rate that boxes of popcorn are sold can be modeled by $S(t)$ where $M(t) = f(t)$ and $S(t) = |g(t)|$ and t is the number of minutes since halftime began. Both $M(t)$ and $S(t)$ are measured in boxes per minute. At time $t = 0$, there are 120 boxes of popcorn.

(**a**) Find $S'(1)$. Using correct units, interpret the meaning of $S'(1)$ in context of the problem.

(**b**) Find the total number of boxes of popcorn sold at the concessions during the first ten minutes of the halftime intermission. Show the computations that lead to your answer.

(**c**) Use a midpoint Riemann sum with the three subintervals indicated in the table to approximate the total number of boxes of popcorn made in the concession stands from $t = 0$ to $t = 10$ minutes.

The problem has been restated.



t	0	1	2	4	6	8	10	13
$f(t)$	7.5	21	27.5	32	45	57	70	100

BC2: The functions f and g are continuous for all $t \geq 0$. Selected values for the function f and a portion of the graph for g are shown above. For $t \geq 0$ the function f is increasing.

Part II: During the halftime intermission at a college football game, popcorn is made and sold at several concession stands throughout the stadium. For $0 \leq t \leq 10$, the rate that boxes of popcorn are made can be modeled by $M(t)$ and the rate that boxes of popcorn are sold can be modeled by $S(t)$ where $M(t) = f(t)$ and $S(t) = |g(t)|$ and t is the number of minutes since halftime began.
At time $t = 0$, there are 120 boxes of popcorn.

(**d**) Write an integral expression for $P(t)$, the number of boxes of popcorn available at the concession stands at time t. Use your answer from part (c) to approximate $P(10)$.

(**e**) For $0 < c < 10$, is there a time t when the rate that boxes of popcorn are being made is equal to the rate that boxes of popcorn are being sold? Explain why or why not.