AP® CALCULUS BC
2007 SCORING GUIDELINES

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

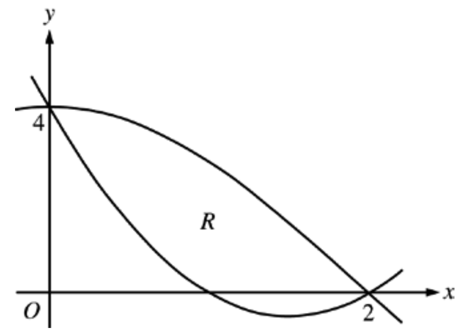
(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$\dfrac{20}{1+x^2} = 2$ when $x = \pm 3$	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \displaystyle\int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961$ or 37.962	2 : { 1 : integrand; 1 : answer
(b) Volume $= \pi \displaystyle\int_{-3}^{3} \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$	3 : { 2 : integrand; 1 : answer
(c) Volume $= \dfrac{\pi}{2} \displaystyle\int_{-3}^{3} \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$ $= \dfrac{\pi}{8} \displaystyle\int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$	3 : { 2 : integrand; 1 : answer

Question 5

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^2 [g(x) - f(x)]\, dx$

$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$

$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$

$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$

$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

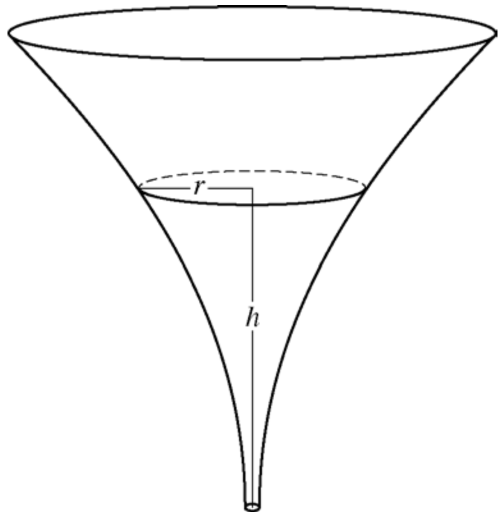
(c) Volume $= \int_0^2 [g(x) - f(x)]^2\, dx$

$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

AP® CALCULUS AB/CALCULUS BC
2016 SCORING GUIDELINES

Question 5



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3 + h^2\right) dh = \frac{1}{200}\left[3h + \frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30 + \frac{1000}{3}\right) - 0\right) = \frac{109}{60}$ in

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi\int_0^{10} \left(\left(\frac{1}{20}\right)\left(3 + h^2\right)\right)^2 dh = \frac{\pi}{400}\int_0^{10}\left(9 + 6h^2 + h^4\right) dh$

$= \frac{\pi}{400}\left[9h + 2h^3 + \frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90 + 2000 + \frac{100000}{5}\right) - 0\right) = \frac{2209\pi}{40}$ in^3

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

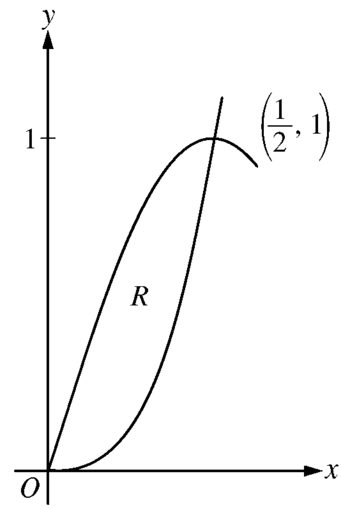
$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec

3 : { 2 : chain rule; 1 : answer }

Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.



(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b) Area $= \int_0^{1/2} (g(x) - f(x))\, dx$

$= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$

$= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$

$= -\frac{1}{8} + \frac{1}{\pi}$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx$

$= \pi\int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

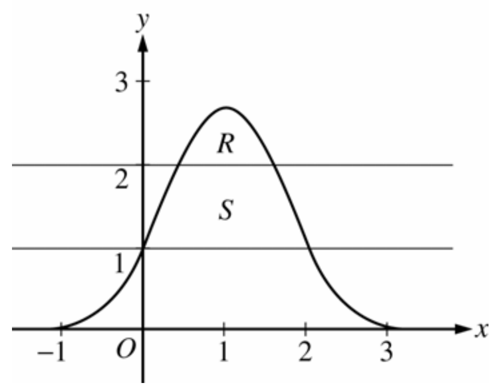
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0,\ 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx - \text{Area of } R$

$= 2.06016 - \text{Area of } R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{constant and limits} \end{cases}$

Question 1

	1 : units in parts (a), (c), and (d)
(a) Volume $= \int_0^{10} A(h)\,dh$ $\approx (2-0)\cdot A(0) + (5-2)\cdot A(2) + (10-5)\cdot A(5)$ $= 2\cdot 50.3 + 3\cdot 14.4 + 5\cdot 6.5$ $= 176.3$ cubic feet	$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$
(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.	1 : overestimate with reason
(c) $\int_0^{10} f(h)\,dh = 101.325338$ The volume is 101.325 cubic feet.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) Using the model, $V(h) = \int_0^h f(x)\,dx.$ $\left.\frac{dV}{dt}\right\|_{h=5} = \left[\frac{dV}{dh}\cdot\frac{dh}{dt}\right]_{h=5}$ $= \left[f(h)\cdot\frac{dh}{dt}\right]_{h=5}$ $= f(5)\cdot 0.26 = 1.694419$ When $h = 5,$ the volume of water is changing at a rate of 1.694 cubic feet per minute.	$3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$