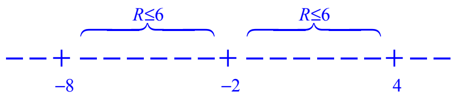
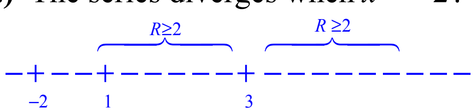
Check for Understanding

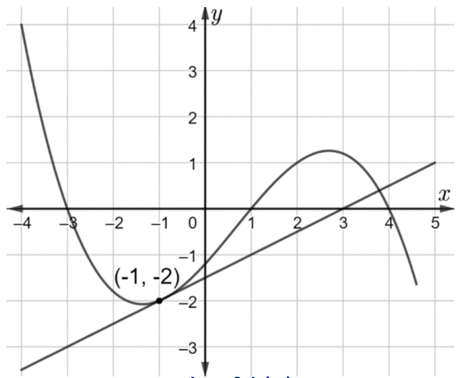
Practice 1: The series $\sum_{n=1}^{\infty} a_n (x+2)^n$ diverges when $x = 4$. Determine if the following statements are **A**lways true, **S**ometimes true or **N**ever true

a.) The series diverges when $x = 5$.  Always True, if $x = 4$ is not in the interval of convergence centered at $x = -2$ then $R \le 6$, so $x = 5$ is not in the interval of convergence.	**b.)** The series converges when $x = -8$. Sometimes true because $x = -8$ could be an endpoint of the interval of convergence.
c.) The series diverges when $x = 0$. Sometimes true because since $R < 6$ then R could be less than 2.	**d.)** The series converges absolutely when $x = -8$. Never true because $x = -8 \Rightarrow \sum_{n=1}^{\infty} a_n (-8+2)^n = \sum_{n=1}^{\infty} a_n (-6)^n$ absolutely convergent $\Rightarrow \sum_{n=1}^{\infty} \left\| a_n (-6)^n \right\| = \sum_{n=1}^{\infty} a_n (6)^n$ converges This is not possible because when $x = 4, \sum_{n=1}^{\infty} a_n (4+2)^n$ diverges

Practice 2: The series $\sum_{n=1}^{\infty} a_n (x-3)^n$ converges when $x = 1$. Determine if the following statements are **A**lways true, **S**ometimes true or **N**ever true

a.) The series diverges when $x = -2$.  Sometimes true because $R \ge 2$ so $x = -2$ could be outside the interval of convergence	**b.)** The series converges when $x = 2$. See part a.) Always true because $R \ge 2$ which includes $x = 2$.
c.) The series diverges when $x = 6$. Sometimes true because R would have to be at least 3 which is possible.	**d.)** The series converges conditionally when $x = 5$. Never true because $x = 5 \Rightarrow \sum_{n=1}^{\infty} a_n (5-3)^n = \sum_{n=1}^{\infty} a_n (2)^n$ which is not an alternating series so there can be no conditional convergence.

Check for Understanding



Graph of $k(x)$

Practice 1: A function k has derivatives of all orders for all values of x. A portion of the graph of k is shown above with the line tangent to the graph of k at $x=-1$. For $n \geq 2$, the nth derivative of $k(x)$ at $x=-1$ is given by:

$$k^{(n)}(-1)=\frac{n!}{n+1}.$$

Find the third degree Taylor polynomial for $k(x)$ about $c=-1$.

$$P_3(x)=k(-1)+k'(-1)(x+1)+\frac{k''(-1)}{2!}(x+1)^2+\frac{k'''(-1)}{3!}(x+1)^3$$

$$P_3(x)=(-2)+\frac{1}{2}(x+1)+\frac{\frac{2!}{2+1}}{2!}(x+1)^2+\frac{\frac{3!}{3+1}}{3!}(x+1)^3$$

$$P_3(x)=-2+\frac{1}{2}(x+1)+\frac{1}{3}(x+1)^2+\frac{1}{4}(x+1)^3$$

Practice 2: The functions f, f', and f'' are each continuous and differentiable. The nth derivative of f is given by $f^{(n)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{n+1}{5}\right)^i$ when $0 \le n \le 3$. Find the third degree Taylor polynomial for $f(x)$ centered around $x = 1$.

$$f(1) = f^{(0)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{0+1}{5}\right)^i = \frac{12}{1-\frac{1}{5}} = \frac{60}{5-1} = 15$$
geometric series $a = 12, r = \frac{1}{5}$

$$f'(1) = f^{(1)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{1+1}{5}\right)^i = \frac{12}{1-\frac{2}{5}} = \frac{60}{5-2} = 20$$
geometric series $a = 12, r = \frac{2}{5}$

$$f''(1) = f^{(2)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{2+1}{5}\right)^i = \frac{12}{1-\frac{3}{5}} = \frac{60}{5-3} = 30$$
geometric series $a = 12, r = \frac{3}{5}$

$$f'''(1) = f^{(3)}(1) = \sum_{i=0}^{\infty} 12\left(\frac{3+1}{5}\right)^i = \frac{12}{1-\frac{4}{5}} = \frac{60}{5-4} = 60$$
geometric series $a = 12, r = \frac{4}{5}$

$$P_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3$$
$$P_3(x) = 15 + (20)(x-1) + \frac{30}{2!}(x-1)^2 + \frac{60}{3!}(x-1)^3 = 15 + 20(x-1) + 15(x-1)^2 + 10(x-1)^3$$

Practice 3: The function g is continuous and has derivatives for all orders at $x = -1$. It is known that $g(-1) = 7$ and for positive values of n, the nth derivative of g at $x = -1$ is defined as the piecewise function given below:

$$g^{(n)}(-1) = \begin{cases} n^2 + 1, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

a.) Find the 5th degree Taylor polynomial, $P_5(x)$, for g centered at $x = -1$.

$$P_5(x) = g(-1) + g'(-1)(x+1) + \frac{g''(-1)}{2!}(x+1)^2 + \frac{g'''(-1)}{3!}(x+1)^3 + \frac{g^{(4)}(-1)}{4!}(x+1)^4 + \frac{g^{(5)}(-1)}{5!}(x+1)^5$$

$$P_5(x) = 7 + (1^2+1)(x+1) + \frac{0}{2!}(x+1)^2 + \frac{(3^2+1)}{3!}(x+1)^3 + \frac{0}{4!}(x+1)^4 + \frac{(5^2+1)}{5!}(x+1)^5$$

$$P_5(x) = 7 + 2(x+1) + \frac{5}{3}(x+1)^3 + \frac{13}{60}(x+1)^5$$

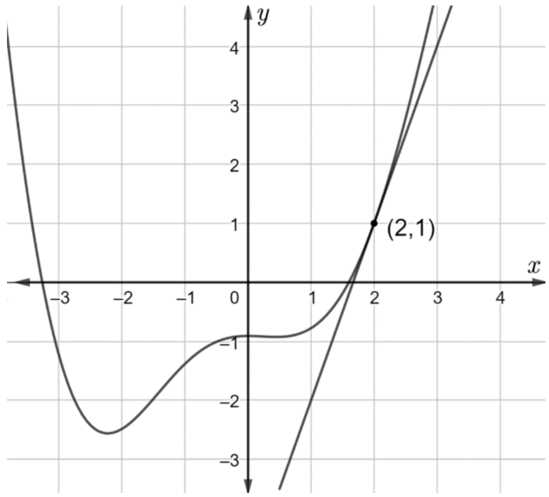
b.) Determine if $P_5(x)$ is increasing or decreasing at $x = -1$. Explain your reasoning.

$$P_5'(x) = 2 + 5(x+1)^2 + \frac{13}{12}(x+1)^4 \Rightarrow P_5'(-1) = 2$$

$P_5(x)$ is increasing at $x = -1$ because $P_5'(-1) = 2 > 0$

AP Exam Practice

AP Practice Problem



Graph of f

A function f has derivatives of all orders for all values of x. A portion of the graph of f is shown above with the line tangent to the graph of f at $x = 2$. Let g be the function defined by $g(x) = 3 + \int_2^x f(t)dt$

(a) Find the second degree Taylor polynomial, $P_2(x)$, for $g(x)$ centered at $x = 2$.

$$g(2) = 3 + \int_2^2 f(t)dt = 3 \qquad g'(2) = f(2) = 1 \qquad g''(2) = f'(2) = 3$$

$$P_2(x) = g(2) + g'(2)(x-2) + \frac{g''(2)(x-2)^2}{2!} = 3 + (x-2) + \frac{3(x-2)^2}{2!} = 3 + (x-2) + \frac{3}{2}(x-2)^2$$

(b) Does $g(x)$ have a local minimum, local maximum, or neither at $x = 2$?

Give a reason for your answer.

$g'(2) = 1 \Rightarrow$ neither because $g'(2) \neq 0$, so $x = 2$ is not a critical point

(c) Consider the geometric series $\sum_{n=1}^{\infty} a_n$ where $a_1 = g'(2)$ and $a_2 = P_2'(x) - 1$.

Find $\sum_{n=1}^{\infty} a_n$ when $x = \frac{13}{6}$.

$$a_1 = g'(2) = 1 \qquad P_2(x) = 3 + (x-2) + \frac{3(x-2)^2}{2!} \qquad a_2 = P_2'(x) - 1 = 3(x-2)$$

$$r = \frac{a_2}{a_1} = \frac{3(x-2)}{1} = 3(x-2) \qquad x = \frac{13}{6} \Rightarrow r = 3\left(\frac{13}{6} - 2\right) = 3\left(\frac{1}{6}\right) = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} a_n = \frac{1}{1 - \frac{1}{2}} = \frac{2}{2-1} = 2$$