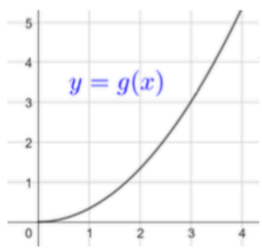
AP Review 15: Arc Length

Arc Length

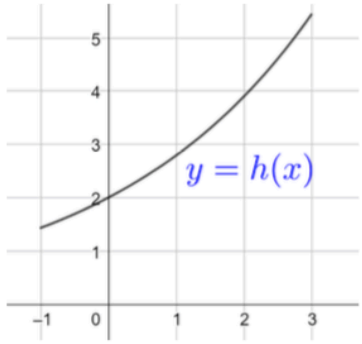
Let the function $y = f(x)$ represent a smooth curve on the interval $[a, b]$, the arc length of f from $x = a$ to $x = b$ is defined by:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Quick Check 1: Consider the function $f(x) = \cos(3x)$. Write, but do not evaluate, an integral expression that gives the arc length of $f(x)$ from 0 to $\frac{2\pi}{3}$



Quick Check 2: A portion of the function $g(x)$ is given above. The arc length of the function $g(x)$ over the interval $[0,3]$ is given by $\int_0^3 \sqrt{1 + \frac{4}{9}x^2}\, dx$. Find $g'(x)$.



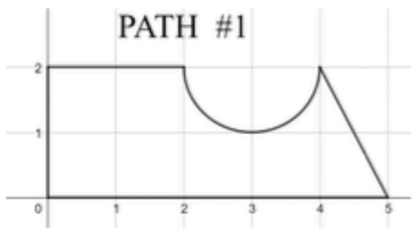
Quick Check 3: A portion of the function $h(x)$ is given above. The arc length of the function $h(x)$ over the interval $[0,2]$ is $\int_0^2 \sqrt{1+\left[\frac{2}{3}e^{\frac{x}{3}}\right]^2}\,dx$. Find the equation of the tangent line to $h(x)$ at $x = 0$.

Quick Check 4: Consider the function $f(x) = \ln(3x^2+2)$. The length of the curve of $f(x)$ from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1+\frac{ax^2}{(3x^2+2)^2}}\,dx$. Find the value of a.

Quick Check 5: Let $y = f(g(2x))$. Write an integral expression that represents the arc length of y from 0 to 5.

Homework

For pages 3-4, you may either try the problems on your own and then correct your work or work alongside the answers. For Pages 5-6, complete the problems without notes and then score yourself as normal!

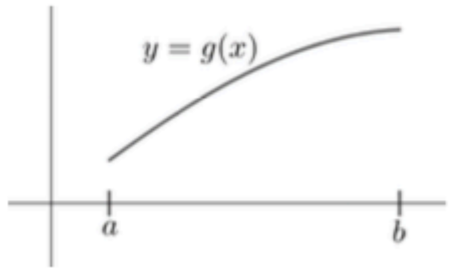


BC 1: Tony and Bryan both start at the origin and walk the closed paths as shown above to end back at the origin. Tony follows Path #1 while Bryan follows Path #2. Path #1 consists of four line segments and a semi circle while Path #2 consists of four line segments and the function $f(x) = e^{\frac{x}{2}}$.

(a) Who took the longer path, Bryan or Tony?

(b) Whose path created the region with the largest area?

(c) Let $P_2(x)$ be the second degree Maclaurin polynomial for $f(x)$. Find $P_2(x)$ and use it to approximate $f(2)$.



BC 4: A portion of the graph for the differentiable function g is given above for $a \leq x \leq b$.

The arc length of g over the interval $[a, b]$ is equal to L where $L = \int_a^b \sqrt{1 + 4\cos^2(2x)}\, dx$.

(**a**) Find $g'(0)$

(**b**) Find $\int g(x)dx$

(**c**) Find the equation of the line tangent to $g(x)$ at $x = \frac{\pi}{6}$

BC Calc 2004 #5 No Calc

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t \to \infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t \to \infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t \to \infty} Y(t)$?

BC Calc 2004 #6 No Calc

Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

(b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

(d) Let G be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for G about $x = 0$.