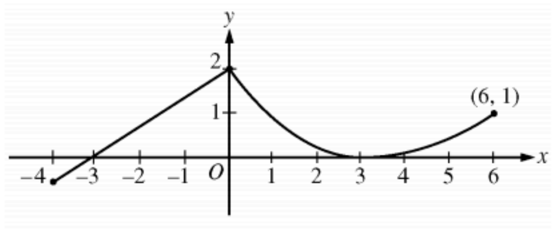
# AP Review 16: Mixed Review 1

AB/BC Calc 2009 Form B #3 No Calc




Graph of $f$

A continuous function $f$ is defined on the closed interval $-4 \le x \le 6$. The graph of $f$ consists of a line segment and a curve that is tangent to the $x$-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function $f$ is twice differentiable, with $f''(x) > 0$.

(a) Is $f$ differentiable at $x = 0$ ? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of $a$, $-4 \le a < 6$, is the average rate of change of $f$ on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

(c) Is there a value of $a$, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value $c$, $a < c < 6$, at which $f'(c) = \frac{1}{3}$ ? Justify your answer.

(d) The function $g$ is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of $g$ concave up? Explain your reasoning.

AB/BC Calc 2009 Form B #6 No Calc

| $t$ (seconds) | 0 | 8 | 20 | 25 | 32 | 40 |
|---|---|---|---|---|---|---|
| $v(t)$ (meters per second) | 3 | 5 | –10 | –8 | –4 | 7 |

The velocity of a particle moving along the $x$-axis is modeled by a differentiable function $v$, where the position $x$ is measured in meters, and time $t$ is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

BC Calc 2009 Form B #6 No Calc (**BC ONLY**)

The function $f$ is defined by the power series

$$f(x) = 1 + (x+1) + (x+1)^2 + \cdots + (x+1)^n + \cdots = \sum_{n=0}^{\infty}(x+1)^n$$

for all real numbers $x$ for which the series converges.

(a) Find the interval of convergence of the power series for $f$. Justify your answer.

(b) The power series above is the Taylor series for $f$ about $x = -1$. Find the sum of the series for $f$.

(c) Let $g$ be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the value of $g\left(-\frac{1}{2}\right)$, if it exists, or explain why $g\left(-\frac{1}{2}\right)$ cannot be determined.

(d) Let $h$ be the function defined by $h(x) = f\left(x^2 - 1\right)$. Find the first three nonzero terms and the general term of the Taylor series for $h$ about $x = 0$, and find the value of $h\left(\frac{1}{2}\right)$.

Homework

As usual, try the problems without notes, then score yourself using the videos and scoring guides. Make corrections in a different color and then fill out the score report form. Email me more your work with corrections. AB Test Takers need to on do the first TWO FRQs. BC Test Takers, need to do the LAST THREE FRQs. You may do the first, if you wish.

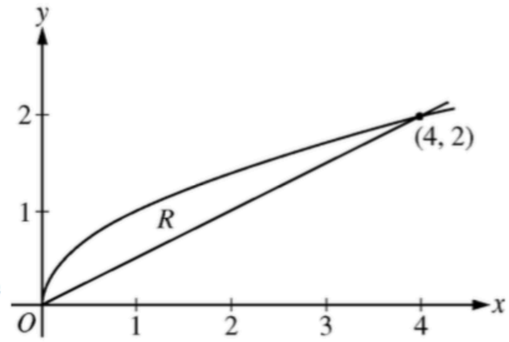
AB Calc 2009 Form B #4 No Calc (**AB ONLY**)

Let $R$ be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

(a) Find the area of $R$.

(b) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when $R$ is rotated about the horizontal line $y = 2$.



AB/BC Calc 2015 #4 No Calc (**AB/BC**)

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\frac{d^2y}{dx^2}$ in terms of $x$ and $y$. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does $f$ have a relative minimum, a relative maximum, or neither at $x = 2$ ? Justify your answer.

(d) Find the values of the constants $m$ and $b$ for which $y = mx + b$ is a solution to the differential equation.

BC Calc 2007 #4 No Calc (**BC ONLY**)

Let $f$ be the function defined for $x > 0$, with $f(e) = 2$ and $f'$, the first derivative of $f$, given by $f'(x) = x^2 \ln x$.

(a) Write an equation for the line tangent to the graph of $f$ at the point $(e, 2)$.

(b) Is the graph of $f$ concave up or concave down on the interval $1 < x < 3$ ? Give a reason for your answer.

(c) Use antidifferentiation to find $f(x)$.

BC Calc 2006 #5 No Calc (**BC ONLY**)

Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

(a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

(b) Is it possible for the $x$-axis to be tangent to the graph of $f$ at some point? Explain why or why not.

(c) Find the second-degree Taylor polynomial for $f$ about $x = -1$.

(d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.