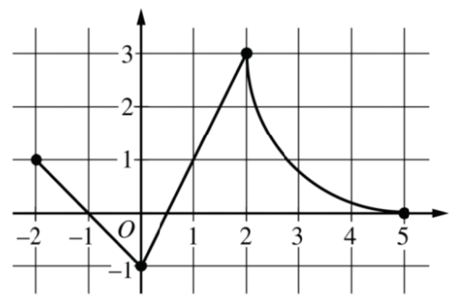
2020 AP Calculus BC: Practice Exam Question #1

WEDNESDAY May 6th, 2020: You will have 25 minutes to complete this problem plus 5 minutes to upload.



Graph of f

1. The continuous function f is defined on the closed interval $-2 \le x \le 5$ and consists of two line segments and a quarter circle centered at the point $(5, 3)$, as shown in the figure above. The function g is given by $g(x) = \int_{-2}^{x} f(t)dt$.

(a) Find the average rate of change of g over the interval $[-2, 5]$.

$$R_{avg} = \frac{g(5)-g(-2)}{5-(-2)} = \frac{g(5)-(0)}{7} = \frac{1}{7}\int_{-2}^{5} f(t)\,dt$$

$$= \frac{1}{7}\left[\frac{1}{2}(1)(1) - \frac{1}{2}(1)\left(\frac{3}{2}\right) + \frac{1}{2}(3)\left(\frac{3}{2}\right) + (3)^2 - \frac{1}{4}\pi(3)^2\right] = \frac{1}{7}\left[\frac{1}{2} - \left(\frac{3}{4}\right) + \left(\frac{9}{4}\right) + (9) - \frac{9}{4}\pi\right]$$

$$= \frac{1}{7}\left[11 - \frac{9}{4}\pi\right]$$

(b) Find $\lim_{x\to -1} \frac{f(x^2)+x}{f'(x)-x}$.

$\lim_{x\to -1}\left[f(x^2)+x\right] = f(1)+(-1) = 1-1 = 0$ $\qquad$ $\lim_{x\to -1}\left[f'(x)-x\right] = f'(-1)-(-1) = -1+1 = 0$

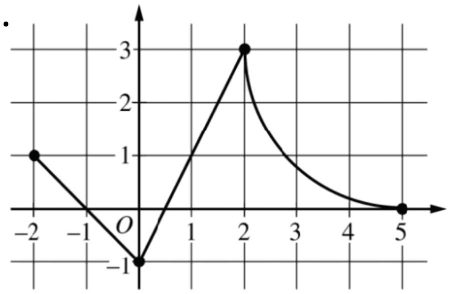
$\lim_{x\to -1} \frac{f(x^2)+x}{f'(x)-x}$ produces the indeterminant form $\frac{0}{0}$ so we can use l'Hospital's Rule.

$$\lim_{x\to -1} \frac{f(x^2)+x}{f'(x)-x} = \underbrace{\lim_{x\to -1} \frac{f'(x^2)(2x)+1}{f''(x)-1}}_{\text{l'Hospital's Rule}} = \frac{f'(1)(-2)+1}{f''(-1)-1} = \frac{(2)(-2)+1}{(0)-1} = 3$$

(c) For $-2 < x < 5$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

g has a point of inflection at $x = 0$ and $x = 2$ because $g'(x) = f(x)$ changes from increasing to decreasing or vice versa.

This is a restatement of the problem.



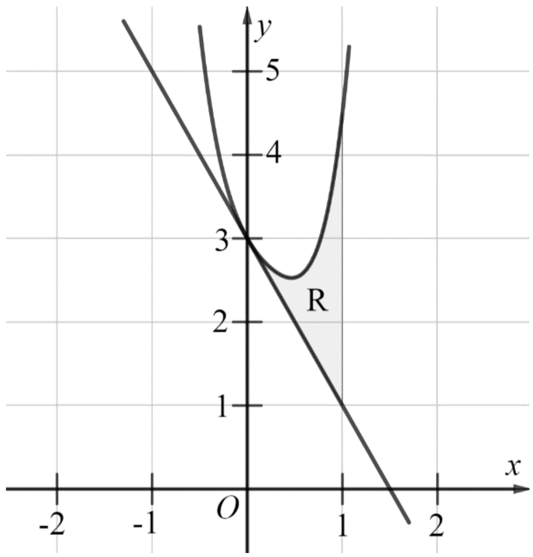
Graph of f

1. The continuous function f is defined on the closed interval $-2 \le x \le 5$ and consists of two line segments and a quarter circle centered at the point $(5, 3)$, as shown in the figure above. The function g is given by $g(x) = \int_{-2}^{x} f(t)dt$.

(d) Evaluate $\int_{2}^{4} f'(6-2x)dx$.

$$\int_{2}^{4} f'(6-2x)dx = -\frac{1}{2}\int_{2}^{4} f'\Big(\underbrace{6-2x}_{u}\Big)\underbrace{(-2dx)}_{du} = -\frac{1}{2}\Big[f(6-2x)\Big]_2^4$$

$$= -\frac{1}{2}\Big[f(-2)-f(2)\Big] = -\frac{1}{2}\Big[(1)-(3)\Big] = 1$$



n	$h^{(n)}(0)$
2	3
3	$-\dfrac{23}{2}$
4	54

A function h has derivatives of all orders for all real numbers x. A portion of the graph of h is shown above, along with the line tangent to the graph of h at $x = 0$. Selected derivatives of h at $x = 0$ are given in the table above. *Let R* be the region bounded by the graphs of h and the line tangent to h at $x = 0$, and the line $x = 1$, as shown in the figure above.

(e) Write the third degree Taylor polynomial for h about $x = 0$.

$$P_3(x) = h(0) + h'(0)x + \frac{h''(0)}{2!}x^2 + \frac{h'''(0)}{3!}x^3$$

$$= 3 - 2x + \frac{3}{2}x^2 - \frac{23}{2(3\cdot 2\cdot 1)}x^3 = 3 - 2x + \frac{3}{2}x^2 - \frac{23}{12}x^3$$

(f) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

$$V = \pi\int_0^1 \left[\left(h(x) - (-2)\right)^2 - \left((-2x+3) - (-2)\right)^2\right]dx$$

(g) Evaluate $\displaystyle\int_1^\infty \frac{1}{x^{p+1}}\,dx$, where $p > 0$.

$$\int_1^\infty \frac{1}{x^{p+1}}dx = \lim_{b\to\infty}\int_1^b \left(x^{(p+1)^{-1}}\right)dx = \lim_{b\to\infty}\int_1^b \left(x^{(-p-1)}\right)dx = \lim_{b\to\infty}\left[-\frac{1}{p}x^{-p}\right]_1^b$$

$$= \lim_{b\to\infty}\left[\left(-\frac{1}{p}b^{-p}\right) - \left(-\frac{1}{p}(1)^{-p}\right)\right] = \left[\left(-\frac{1}{p}(0)\right) - \left(-\frac{1}{p}(1)^{-p}\right)\right] = \frac{1}{p}$$

2020 AP Calculus BC: Practice Exam Question #2 Solutions Day 29

x	1	3	8	9
$f(x)$	6	4	5	2
$f'(x)$	2	-2	3	-1

2. The function f is twice differentiable with selected values given in the table above.

(**a**) Let $g(x) = \dfrac{x^2}{f(x)}$. Find $g'(3)$.

$$g'(x) = \frac{f(x)(2x) - f'(x)(x^2)}{(f(x))^2}$$

$$g'(3) = \frac{f(3)(2(3)) - f'(3)(3^2)}{(f(3))^2} = \frac{(4)(6)-(-2)(9)}{(4)^2} = \frac{24+18}{16} = \frac{42}{16} = \frac{21}{8}$$

(**b**) Use a left Riemann sum with the three subintervals indicated in the table above to approximate the average value of $f(x)$ over the interval $[1, 9]$.

$$Avg = \frac{1}{8}\int_1^9 f(x)\,dx \approx \frac{1}{8}\left[f(1)(2) + f(3)(5) + f(8)(1)\right]$$

$$= \frac{1}{8}\left[(6)(2)+(4)(5)+(5)(1)\right] = \frac{1}{8}\left[12+20+5\right] = \frac{37}{8}$$

(**c**) Evaluate $\int_3^8 x f''(x)\,dx$.

$$\int \underbrace{x}_{u}\, \underbrace{f''(x)\,dx}_{dv} = x f'(x) - \int f'(x)\,dx = x f'(x) - f(x)$$

$$u = x \Rightarrow du = dx$$
$$dv = f''(x)\,dx \Rightarrow v = f'(x)$$

$$\int_3^8 x f''(x)\,dx = \left[x f'(x) - f(x)\right]_3^8 = \left[(8)f'(8) - f(8)\right] - \left[(3)f'(3) - f(3)\right]$$

$$= \left[(8)(3)-(5)\right] - \left[(3)(-2)-(4)\right] = [19] - [-10] = 29$$

2020 AP Calculus BC: Practice Exam Question #2 Solutions Day 29

x	1	3	8	9
$f(x)$	6	4	5	2
$f'(x)$	2	-2	3	-1

2. The function f is twice differentiable with selected values given in the table above.

(**d**) Let $H(x) = \int_1^{x^2} f(t)dt$. Find $H'(x)$ and $H''(x)$. Explain why H could not have a relative extremum or a point of inflection at $x = 3$.

$H'(x) = f(x^2)(2x)$ $H''(x) = f(x^2)(2) + f'(x^2)(2x)^2$

$H'(3) = f(3^2)(2(3)) = (2)(6) = 12$ $H''(x) = (2)(2) + (-1)(6)^2 = 4 - 36 = -32$

H could not have a relative extremum at $x = 3$ because $H'(3) \neq 0$ or undefined.

H could not have a point of inflection at $x = 3$ because $H''(3) \neq 0$ or undefined.

(**e**) Let $f(a) = \sum_{n=0}^{\infty} ar^n$ where a and r are constants and $5 \leq a \leq 8.$. Find the value of r when $a = 8$.

$\sum_{n=0}^{\infty} 8r^n = f(8) = 5$ $\frac{8}{1-r} = 5 \Rightarrow 8 = 5 - 5r \Rightarrow -5r = 3 \Rightarrow r = -\frac{3}{5}$

2020 AP Calc BC Practice Exam Question #2B

a) $f(x) = f(6) + \int_6^x f'(t)\,dt = -1 + \int_6^x f'(t)\,dt$

Minis at EPs or CPs

EPs: $x = -5$

$f(-5) = -1 + \int_6^{-5} f(t)\,dt = -1 + 2 + 2\pi - \frac{15}{2}$

$x = 10$

$f(10) = -1 + \int_6^{10} f(t)\,dt = -1 + 4 = 3$

CPs (but only care about rel. min):

$x = 6$

$f(6) = -1$

Min value is ~~f(10) = 3~~ $f(6) = -1$

b) $g(x) = \sin(3 - f'(x))$

$g'(x) = \cos(3 - f'(x)) \cdot [-f''(x)]$

y-value on graph ↗ slope on graph ↗

$g'(6) = \cos(3 - f'(6))(-f''(6))$

$g'(6) = \cos(3 - 0)(-2) = \cos(3)(-2)$

c)

(x, y)	Δx	$\frac{dy}{dx}$	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	$(x + \Delta x,\ y + \Delta y)$
$(5, 6)$	-1	$8/5$	$-8/5$	$(4,\ 6 - 8/5)$
$(4,\ 6 - 8/5)$	-1	$11/10$	$-11/10$	$(3,\ 6 - 8/5 - 11/10)$

to go from $x = 5$ to $x = 3$ in 2 steps

$\Delta x = -1$

$\frac{3-5}{2} = -1$

$\frac{dy}{dx} = \frac{y - f'(x)}{x}$

$\left.\frac{dy}{dx}\right|_{(5,6)} = \frac{6 - f'(5)}{5} = \frac{6 - (-2)}{5} = \frac{8}{5}$ (↖ Point on graph)

$\left.\frac{dy}{dx}\right|_{(4,\ 6 - 8/5)} = \frac{6 - 8/5 - f'(4)}{4} = \frac{6 - 8/5}{4} = \frac{11}{10}$

$h(3) \approx 6 - 8/5 - 11/10$

All terms +

d) $\int_0^{10} |v(t)|\,dt = 2\pi + 2 + 4$ meters

$\int_0^{10} |v(t)|\,dt \Rightarrow$ is the total distance traveled by the particle in meters from 0–10 seconds.

e) $K(x) = 2x - \int_6^x f(t)\,dt$

$K(6) = 12 - \int_6^6 f(t)\,dt = 12$

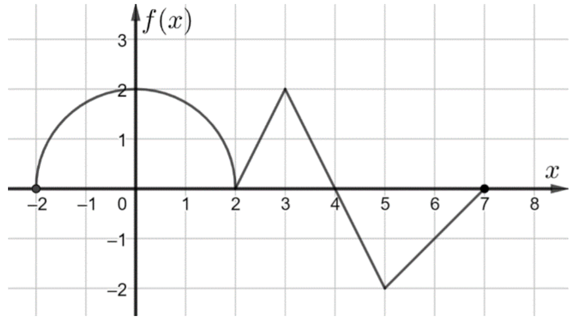
$K'(x) = 2 - f(x) \rightarrow K'(6) = 2 - f(6) = 2 - (-1) = 3$

$K''(x) = -f'(x) \rightarrow K''(6) = -f'(6) = -(0) = 0$

$K'''(x) = -f''(x) \rightarrow K'''(6) = -f''(6) = -2$

$T_3(x) = 12 + 3(x-6) + \frac{0(x-6)^2}{2!} - \frac{2(x-6)^3}{3!}$

5 for 5: Calculus BC Day 1 Solutions



The function f is continuous on the interval $[-2, 7]$ and consists of three line segments and a semi circle as shown in the figure above. The function g is defined by $g(x) = \int_{-2}^{x^2} f(t)dt$.

BC1: Let $h(x) = f(5x - 9)$. Find $h'(3)$.

$$h'(x) = f'(5x - 9)(5) \Rightarrow h'(3) = f'(5(3) - 9)(5) = 5f'(6) = 5(1) = 5$$

BC2: Evaluate $\int_{-1}^{0} [f'(3 - 2x) - 4]\, dx$.

$$\int_{-1}^{0}\left[f'(3-2x)-4\right]\, dx = -\frac{1}{2}\int_{-1}^{0}\left[f'\big(\underbrace{3-2x}_{u}\big)\right](-2dx) - \int_{-1}^{0}[4]\, dx$$

$$= -\frac{1}{2}\left[f(3-2x)\right]_{-1}^{0} - \left[4x\right]_{-1}^{0} = -\frac{1}{2}\left[f(3)-f(5)\right] - \left[-4(-1)\right] = -\frac{1}{2}\left[(2)-(-2)\right] - [4] = -6$$

BC3: Write the 2nd degree Taylor polynomial for g about $x = 2$.

$$g(2) = \int_{-2}^{4} f(t)dt = \left[\frac{1}{2}\pi(2)^2 + \frac{1}{2}(2)(2)\right] = 2\pi + 2$$

$$g'(x) = f(x^2)(2x) \Rightarrow g'(2) = f(4)(4) = (0)(4) = 0$$

$$g''(x) = f(x^2)(2) + f'(x^2)(2x)^2 \Rightarrow g''(2) = f(4)(2) + f'(4)(4)^2 = 16f'(4) = 16(-2) = -32$$

$$P_2(x) = g(2) + g'(2)(x-2) + \frac{g''(2)}{2!}(x-2)^2 = (2\pi + 2) + (0)(x-2) + \frac{-32}{2!}(x-2)^2$$

$$= (2\pi + 2) - 16(x-2)^2$$

t seconds	0	1	4	6
$P(t)$ people per second	8	3	5	10

For $0 \leq t \leq 6$ seconds, people enter a school at the rate $P(t)$, measured in people per second.

BC4: Approximate $P'(5)$. Using correct units, interpret the meaning of $P'(5)$ in the context of the problem.

$$P'(5) \approx \frac{P(6) - P(4)}{6 - 4} = \frac{(10) - (5)}{6 - 4} = \frac{5}{2}$$

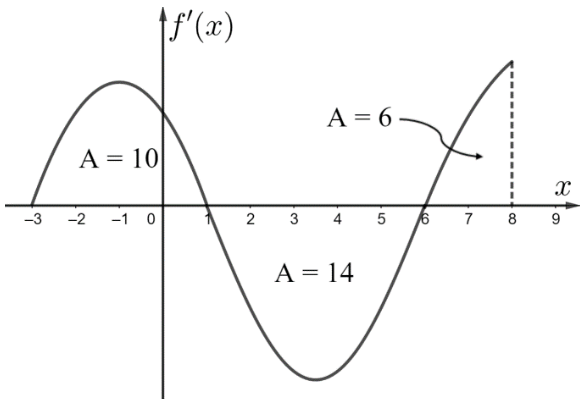
The rate people enter a school is changing at a rate of $P'(5)$ people per second per second at $t = 5$ seconds.

BC5: Use a left Riemann sum with the three subintervals indicated by the table above to approximate $\frac{1}{6}\int_0^6 P(t)dt$. Using correct units, interpret the meaning of $\frac{1}{6}\int_0^6 P(t)dt$ in the context of the problem.

$$\frac{1}{6}\int_0^6 P(t)dt \approx \frac{1}{6}\left[P(0)(1) + P(1)(3) + P(4)(2)\right] = \frac{1}{6}\left[(8)(1) + (3)(3) + (5)(2)\right] = \frac{27}{6} = \frac{9}{2}$$

$\frac{1}{6}\int_0^6 P(t)dt$ is the average rate people enter a school, in people per second over the interval $t = 0$ to $t = 6$ seconds.

5 for 5: Calculus BC Day 2 Solutions



x	1	4	6	9
$g(x)$	3	1	0	-1
$g'(x)$	2	0	1	3

A portion of the graph of f', the derivative of the twice differentiable function f, is shown in the figure above. The areas of the regions bounded by the graph of f' and the x axis are labeled. It is known that $f(1) = -2$.

The function g is twice differentiable. Selected values of g and g' are shown in the table above.

BC1: Find all values of x in the open interval $-3 < x < 8$ for which the graph of f has horizontal tangent line. For each value of x, determine whether f has a relateive minimum, relative maximum, or neither a minimum nor a maximum at the x value. Justify your answers.

horizontal tangent line $\Rightarrow$ $f'(x) = 0 \Rightarrow$ $x = 1, 6$

At $x = 1$ there is a relative maximum because $f'(x)$ changes from positive to negative.

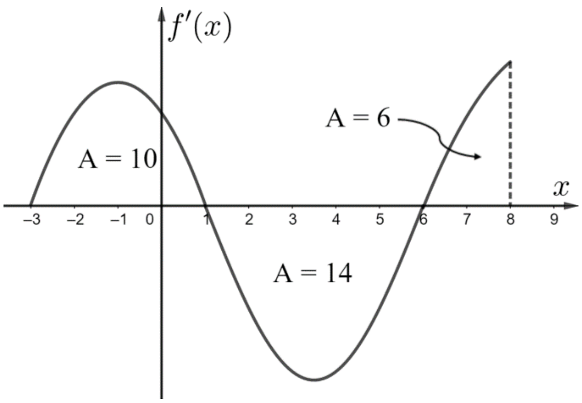
At $x = 6$ there is a relative minimum because $f'(x)$ changes from negative to positive.

BC2: Find the minimum value of f on the closed interval $[-3, 8]$. Justify your answer..

relative minimum candidates: $x = 6$ endpoints: $x = -3, 8$

x	$f(x)$
-3	$-2-\int_{-3}^{1} f'(x)\,dx = -2-(10) = -12$
6	$-2+\int_{1}^{6} f'(x)\,dx = -2-(14) = -16$
8	$-2+\int_{1}^{8} f'(x)\,dx = -2-(14)+6 = -10$

5 for 5: Calculus BC Day 2



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The function g is twice differentiable. Selected values of g and g' are shown in the table above.

BC3: Let $h(x) = \dfrac{e^{g(x)}}{3x}$. Find $h'(6)$.

$$h'(x) = \frac{(3x)\left(e^{g(x)}\right)\left(g'(x)\right) - (3)\left(e^{g(x)}\right)}{(3x)^2}$$

$$h'(6) = \frac{(18)\left(e^{g(6)}\right)\left(g'(6)\right) - (3)\left(e^{g(6)}\right)}{(18)^2} = \frac{(18)\left(e^{0}\right)(1) - (3)\left(e^{0}\right)}{(18)^2} = \frac{(18) - (3)}{(18)^2} = \frac{15}{(18)^2} = \frac{5}{108}$$

BC4: For $t \geq 0$, a particle moves along a straight path with velocity $v(t) = f'(t)$. Find the total distance traveled by the particle from $t = 1$ to $t = 8$.

$$T = \int_1^8 \left|v(t)\right| dt = \int_1^8 \left|f'(t)\right| dt = 14 + 6 = 20$$

BC5: Evaluate $\int_1^9 xg''(x)dx$.

$$\int \underbrace{x}_{u}\,\underbrace{g''(x)\,dx}_{dv} = x\,g'(x) - \int g'(x)\,dx \qquad u = x \Rightarrow du = dx$$

$$dv = g''(x)\,dx \Rightarrow v = g'(x)$$

$$= x\,g'(x) - g(x) + C$$

$$\int_1^9 xg''(x)\,dx = \left[x\,g'(x) - g(x)\right]_1^9 = \left[\left((9)g'(9) - g(9)\right) - \left((1)g'(1) - g(1)\right)\right]$$

$$= \left[\left((9)(3) - (-1)\right) - \left((2) - (3)\right)\right] = \left[(28) - (-1)\right] = 29$$

5 for 5 Calculus BC Day 3

BC1

$g(3)=5$ $\quad y-5=-2(x-3)$

$g'(3)=-2$ $\quad y=-2(x-3)+5$

when $x=2$ $\quad \boxed{y=-2(2-3)+5}$

BC2

$$\lim_{x\to -1} \frac{\int_{-3}^{x^2} f(t)\,dt}{x^3+1} \Rightarrow \frac{\int_{-3}^{1} f(t)\,dt}{0} \Rightarrow \frac{0}{0} \text{ L'Hopital's Rule!}$$

deriv

$$\lim_{x\to -1} \frac{\int_{-3}^{x^2} f(t)\,dt}{x^3+1} \overset{H}{=} \lim_{x\to -1} \frac{f(x^2)\cdot 2x}{3x^2} = \frac{f(1)\cdot(-2)}{3} = \frac{-2}{3}$$

BC3

$$p(x) = \begin{cases} f(x)g'(x) & x<3 \\ 4f'(x-3) & x\geq 3 \end{cases}$$

1. $\lim_{x\to 3^-} p(x) = \lim_{x\to 3^+} p(x)$

→ slope of graph

$\lim_{x\to 3^-} f(x)g'(x) = \lim_{x\to 3^+} 4f'(x-3)$

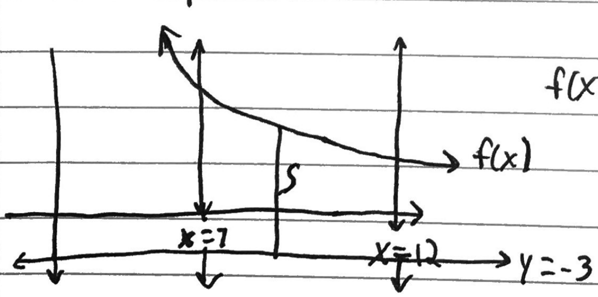
$(-1)(-2) = 4(\frac{1}{2})$

$2=2$ ✓

2. $p(3)=4f'(x-3)=2$ ✓
3. $p(3)=\lim_{x\to 3} p(x)=2$ ✓

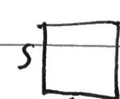
$\therefore$ $p(x)$ is continuous at $x=3$.

BC4



$f(x)=\frac{a}{x^{3p+2}}$

No matter what a and p are f(x) decreases as $x\to\infty$



$A=s^2$ $\quad s=f(x)-(-3)=f(x)+3$

$A=[f(x)+3]^2$

Cross Sections are squares

$V=\int_a^b \text{Area}\,dx=\int_7^{12}[f(x)+3]^2\,dx$

BC5

$\sum_{n=7}^{\infty} b_n = \sum_{n=7}^{\infty} \frac{a}{x^{3p+2}} = a\sum_{n=7}^{\infty} \frac{1}{x^{3p+2}}$ this is a p-series; p-series converge if $b>1$

↳ $\frac{1}{x^b}$

$3p+2>1$

$3p>-1$

$\boxed{p>-\frac{1}{3}}$ → this makes the power >1