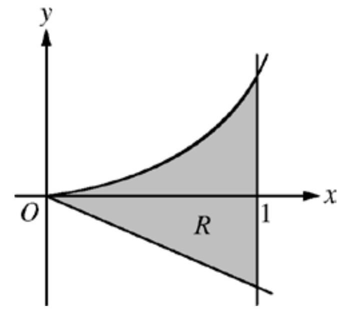
Question 5

Solution	Scoring
(a) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2}$ $f'(3) = \dfrac{(-3)(5)}{(18-21+5)^2} = -\dfrac{15}{4}$	2 : $f'(3)$
(b) $f'(x) = \dfrac{-3(4x-7)}{\left(2x^2-7x+5\right)^2} = 0 \Rightarrow x = \dfrac{7}{4}$ The only critical point in the interval $1 < x < 2.5$ has x-coordinate $\dfrac{7}{4}$. f' changes sign from positive to negative at $x = \dfrac{7}{4}$. Therefore, f has a relative maximum at $x = \dfrac{7}{4}$.	2 : { 1 : x-coordinate; 1 : relative maximum with justification }
(c) $\int_5^\infty f(x)\,dx = \lim_{b\to\infty}\int_5^b \dfrac{3}{2x^2-7x+5}\,dx = \lim_{b\to\infty}\int_5^b \left(\dfrac{2}{2x-5} - \dfrac{1}{x-1}\right)dx$ $= \lim_{b\to\infty}\Big[\ln(2x-5) - \ln(x-1)\Big]_5^b = \lim_{b\to\infty}\left[\ln\left(\dfrac{2x-5}{x-1}\right)\right]_5^b$ $= \lim_{b\to\infty}\left[\ln\left(\dfrac{2b-5}{b-1}\right) - \ln\left(\dfrac{5}{4}\right)\right] = \ln 2 - \ln\left(\dfrac{5}{4}\right) = \ln\left(\dfrac{8}{5}\right)$	3 : { 1 : antiderivative; 1 : limit expression; 1 : answer }
(d) f is continuous, positive, and decreasing on $[5, \infty)$. The series converges by the integral test since $\int_5^\infty \dfrac{3}{2x^2-7x+5}\,dx$ converges. — OR — $\dfrac{3}{2n^2-7n+5} > 0$ and $\dfrac{1}{n^2} > 0$ for $n \ge 5$. Since $\lim_{n\to\infty}\dfrac{\frac{3}{2n^2-7n+5}}{\frac{1}{n^2}} = \dfrac{3}{2}$ and the series $\sum_{n=5}^{\infty}\dfrac{1}{n^2}$ converges, the series $\sum_{n=5}^{\infty}\dfrac{3}{2n^2-7n+5}$ converges by the limit comparison test.	2 : answer with conditions

Question 5

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

(a) Area $= \int_0^1 \left(xe^{x^2} - (-2x) \right) dx$

$$= \left[\frac{1}{2} e^{x^2} + x^2 \right]_{x=0}^{x=1}$$

$$= \left(\frac{1}{2} e + 1 \right) - \frac{1}{2} = \frac{e+1}{2}$$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \pi \int_0^1 \left[\left(xe^{x^2} + 2 \right)^2 - (-2x + 2)^2 \right] dx$

3 : { 2 : integrand; 1 : limits and constant }

(c) $y' = \frac{d}{dx}\left(xe^{x^2} \right) = e^{x^2} + 2x^2 e^{x^2} = e^{x^2}\left(1 + 2x^2 \right)$

Perimeter $= \sqrt{5} + 2 + e + \int_0^1 \sqrt{1 + \left[e^{x^2}\left(1 + 2x^2 \right) \right]^2}\, dx$

3 : { 1 : $y' = e^{x^2}\left(1 + 2x^2\right)$; 1 : integral; 1 : answer }