AP Review 18: Mixed Review 3

Announcements: Please attend Monday at 3PM for our last Live Class!
Office hours this weekend: 2-4PM Saturday. 2-4PM Sunday (during this time, 2-2:30PM is devoted to scoring the final AP practice FRQ exam).

BC Calc 2017 #5 Non Calc

5. Let f be the function defined by $f(x) = \dfrac{3}{2x^2 - 7x + 5}$.

(a) Find the slope of the line tangent to the graph of f at $x = 3$.

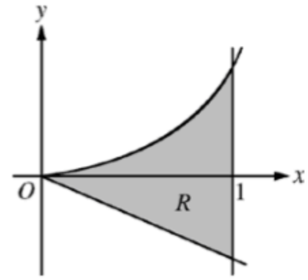
(b) Find the x-coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that $\dfrac{3}{2x^2 - 7x + 5} = \dfrac{2}{2x - 5} - \dfrac{1}{x - 1}$, evaluate $\int_5^\infty f(x)\, dx$ or show that the integral diverges.

(d) Determine whether the series $\displaystyle\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$ converges or diverges. State the conditions of the test used for determining convergence or divergence.

BC Calc 2014 #5 No Calc (**BC Only**)

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.



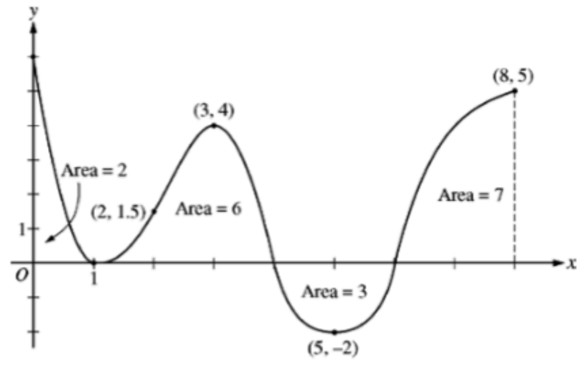
(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

AB/BC Calc 2013 #4 No Calc (**AB/BC**)

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

AB/BC Calc 2011 #5 No Calc (**AB/BC**)

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

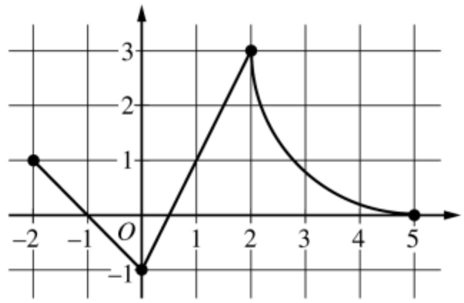
(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

Homework (AB Test Takers)

As usual, try the official FRQs without notes, then score yourself using the videos and scoring guides. Make corrections in a different color and then fill out the score report form. Email me more your work with corrections. I am also including a series of other worksheets. You need to choose at least ONE to do. If you do extra worksheets, I will count them as assignment replacers! In your email subject, please include “Extra Pages Included.”

AB/BC Calc 2019 #3 No Calc



Graph of f

3. The continuous function f is defined on the closed interval $-6 \le x \le 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

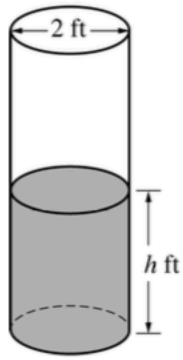
(a) If $\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

(b) Evaluate $\int_{3}^{5} (2f'(x) + 4)\,dx$.

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

(d) Find $\lim\limits_{x \to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x}$.

AB/BC Calc 2019 #4 No Calc



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

AB/BC Calc 2019 #6 No Calc

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

   (a) Find $h'(2)$.

   (b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

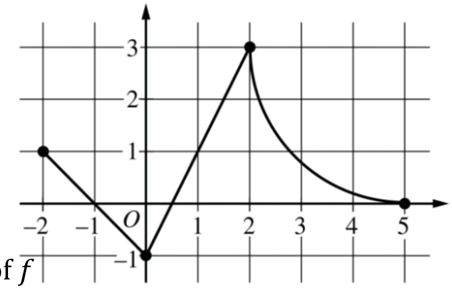
   (c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

   (d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

Sample Question 1

Allotted time: 25 minutes (plus 5 minutes to submit)

t (hours)	0	0.3	1	2.8	4
$v_p(t)$ (meters per hour)	0	55	−29	55	48



Graph of f

The velocity of a particle, P, moving along the x-axis is given by the differentiable function $v_P(t)$, where $v_P(t)$ is measured in meters per hour and t is measured in hours. Selected values of $v_P(t)$ are shown in the table above. Particle P is at the origin at time $t = 0$. The acceleration of particle P, $a_P(t)$, at $t = 1$ is known to be $a_P(1) = -10$.

Also, the continuous function f is defined on the closed interval $-6 \leq t \leq 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3 - \sqrt{5})$ is on the graph of f.

(a) Find $\frac{d}{dt}[f(t) \cdot v_P(t)]|_{t=1}$

(b) Use a trapezoidal sum with the three subintervals $[0, 0.3]$, $[0.3, 1]$, and $[1, 2.8]$ to approximate the value of $\int_0^{2.8} v_p(t)\, dt$.

(c) If $\int_{-6}^{5} f(t)\, dt = 7$, find the value of $\int_{-6}^{-2} f(t)\, dt$. Show the work that leads to your answer.

(d) Evaluate $\int_3^5 (2f'(t) + 4) dt$.

(e) The function g is given by $g(t) = \int_{-2}^{t} f(x)\, dx$. Find the absolute maximum of g on the interval $-2 \leq x \leq 5$. Justify your answer.

(f) Using $g(t)$ from part (e), is the rate of change in g increasing or decreasing at $t = 3$? Explain your reasoning.

(g) Find $\lim_{t \to 1} \frac{e^t - 3f(t)}{v_P(t) - \cos(\pi t)}$.

Sample Question 1

Allotted time: 25 minutes (plus 5 minutes to submit)

Functions f, g, and h are twice-differentiable functions with $g(5) = h(5) = 1$. The line $y = 1 - \frac{5}{3}(x - 5)$ is tangent to both the graph of g at $x = 5$ and the graph of h at $x = 5$.

(a) Find $g'(5)$.

(b) Let b be the function given by $b(x) = 2x^2 g(x)$. Write an expression for $b'(x)$. Find $b'(5)$.

(c) Let w be the function given by $w(x) = \dfrac{3h(x) - x}{2x + 1}$. Write an expression for $w'(x)$. Find $w'(5)$.

(d) Let $M(x) = \dfrac{d}{dx}\left[\displaystyle\int_0^{2x} g(t)\,dt\right]$. Write an expression for $M'(x)$. Find $M'(2.5)$.

(e) Let $M(x) = \dfrac{d}{dx}\left[\displaystyle\int_0^{2x} g(t)\,dt\right]$. It is known that $c = 2.5$ satisfies the conclusion of the Mean Value Theorem applied to $M(x)$ on the interval $1 \le x \le 4$. Use $M'(2.5)$ to find $g(8) - g(2)$.

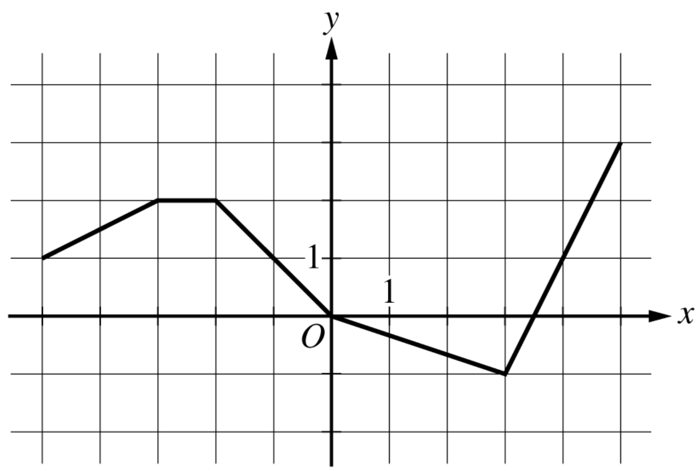
(f) The function g satisfies $g(x) = \dfrac{x + 5\cos\left(\frac{1}{5}\pi x\right)}{3 - \sqrt{f(x)}}$ for $x \neq 5$. It is known that $\lim\limits_{x \to 5} g(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x \to 5} g(x)$ to find $f(5)$ and $f'(5)$. Show the work that leads to your answers.

(g) It is known that $h(x) \le g(x)$ for $4 < x < 6$. Let k be a function satisfying $h(x) \le k(x) \le g(x)$ for $4 < x < 6$. Is k continuous at $x = 5$? Justify your answer.

Sample Question 2

Allotted time: 15 minutes (plus 5 minutes to submit)

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

Let f be the function defined by $f(x) = \sin(\pi x) + \ln(2 - x)$.

Let g be a twice differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = 1$.

(b) Let k be the function defined by $k(x) = h(f(x) + 2)$. Find $k'(1)$.

(c) Evaluate $\int_{-5}^{-1} g'(x)dx$.

(d) Rewrite $\lim_{n\to\infty} \sum_{k=1}^{n} \left(h\left(-1 + \frac{5k}{n}\right)\right)\frac{5}{n}$ as a definite integral in terms of $h(x)$ with a lower bound of $x = -1$.

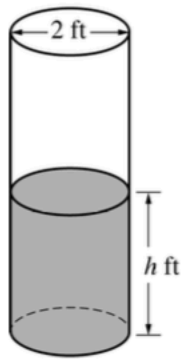
Evaluate the definite integral.

(e) What is the fewest number of horizontal tangents $g(x)$ has on the interval $-5 < x < 0$? Justify your answer.

Homework (BC Test Takers)

As usual, try the official FRQs without notes, then score yourself using the videos and scoring guides. Make corrections in a different color and then fill out the score report form. Email me more your work with corrections. I am also including a series of other worksheets. You need to choose at least ONE to do. If you do extra worksheets, I will count them as assignment replacers! In your email subject, please include “Extra Pages Included.”

BC Calc 2019 #4 No Calc (**BC ONLY**)



4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

   (b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

BC Calc 2019 #5 No Calc (**BC ONLY**)

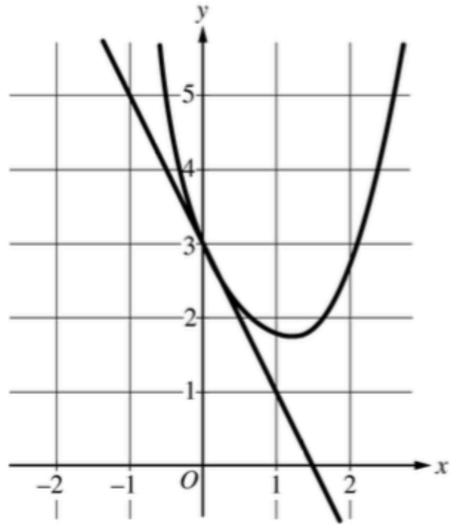
5. Consider the family of functions $f(x) = \dfrac{1}{x^2 - 2x + k}$, where k is a constant.

   (a) Find the value of k, for $k > 0$, such that the slope of the line tangent to the graph of f at $x = 0$ equals 6.

   (b) For $k = -8$, find the value of $\int_0^1 f(x)\, dx$.

   (c) For $k = 1$, find the value of $\int_0^2 f(x)\, dx$ or show that it diverges.

BC Calc 2019 #6 No Calc (**BC**)



n	$f^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

6. A function f has derivatives of all orders for all real numbers x. A portion of the graph of f is shown above, along with the line tangent to the graph of f at $x = 0$. Selected derivatives of f at $x = 0$ are given in the table above.

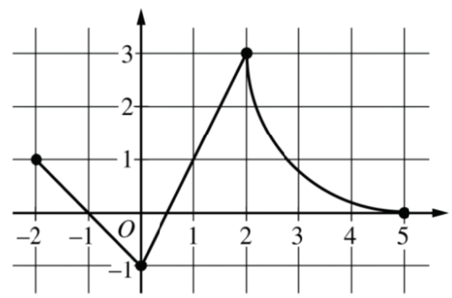
(a) Write the third-degree Taylor polynomial for f about $x = 0$.

(b) Write the first three nonzero terms of the Maclaurin series for e^x. Write the second-degree Taylor polynomial for $e^x f(x)$ about $x = 0$.

(c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Use the Taylor polynomial found in part (a) to find an approximation for $h(1)$.

2020 AP Calculus BC: Practice Exam Question #1

WEDNESDAY May 6th, 2020: You will have 25 minutes to complete this problem plus 5 minutes to upload.



Graph of f

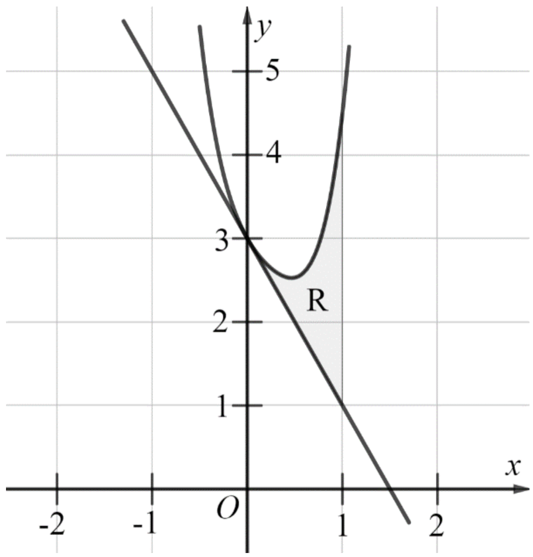
1. The continuous function f is defined on the closed interval $-2 \leq x \leq 5$ and consists of two line segments and a quarter circle centered at the point $(5, 3)$, as shown in the figure above. The function g is given by $g(x) = \int_{-2}^{x} f(t)dt$.

(a) Find the average rate of change of g over the interval $[-2, 5]$.

(b) Find $\lim_{x \to -1} \dfrac{f(x^2) + x}{f'(x) - x}$.

(c) For $-2 < x < 5$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(d) Evaluate $\int_{2}^{4} f'(6 - 2x)dx$.



n	$h^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

A function h has derivatives of all orders for all real numbers x. A portion of the graph of h is shown above, along with the line tangent to the graph of h at $x = 0$. Selected derivatives of h at $x = 0$ are given in the table above. Let R be the region bounded by the graphs of h, the line tangent to h at $x = 0$, and the line $x = 1$, as shown in the figure above.

(e) Write the third degree Taylor polynomial for h about $x = 0$.

(f) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(g) Evaluate $\displaystyle\int_1^{\infty} \frac{1}{x^{p+1}}\,dx$, where $p > 0$.

2020 AP Calculus BC: Practice Exam Question #2

MONDAY May 4th, 2020: You will have 15 minutes to complete this problem plus 5 minutes to upload.

x	1	3	8	9
$f(x)$	6	4	5	2
$f'(x)$	2	-2	3	-1

2. The function f is twice differentiable with selected values given in the table above.

(a) Let $g(x) = \dfrac{x^2}{f(x)}$. Find $g'(3)$.

(b) Use a left Riemann sum with the three subintervals indicated in the table above to approximate the average value of $f(x)$ over the interval $[1, 9]$.

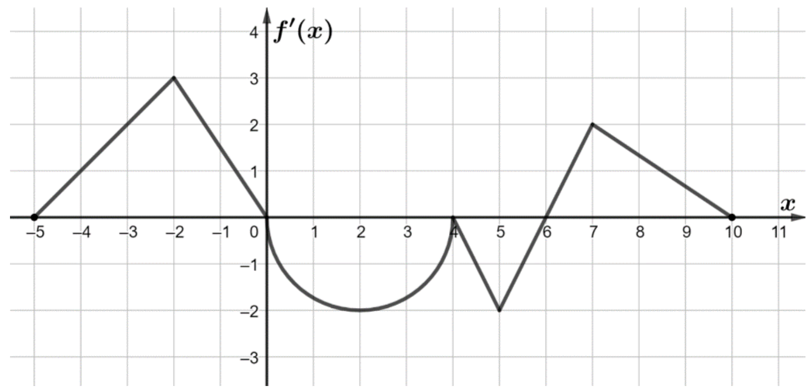
(c) Evaluate $\int_3^8 xf''(x)dx$.

(d) Let $H(x) = \int_1^{x^2} f(t)dt$. Find $H'(x)$ and $H''(x)$. Explain why H could not have a relative extremum or a point of inflection at $x = 3$.

(e) Let $f(a) = \sum_{n=0}^{\infty} ar^n$ where a and r are constants and $5 \leq a \leq 8$. Find the value of r when $a = 8$.

2020 AP Calculus BC: Practice Exam Question #2B

THURSDAY May 7th, 2020: You will have 15 minutes to complete this problem plus 5 minutes to upload.

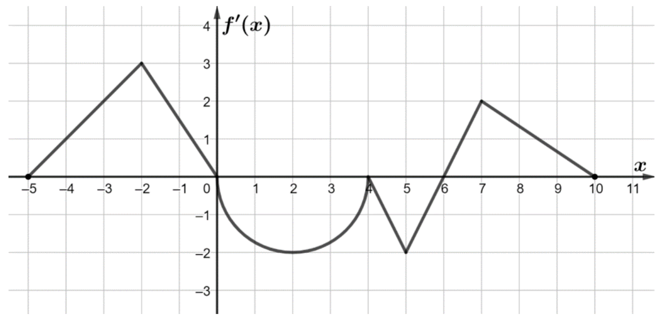


2. The graph of f', the derivative of the differentiable function f, consists of five linear segments and a semi circle and is shown in the figure above. The functions f and f' are defined on the closed interval $[-5, 10]$. It is known that $f(6) = -1$.

(**a**) Find the minimum value of f on the closed interval $-5 \le x \le 10$. Justify your answer.

(**b**) Let $g(x) = \sin\left(3 - f'(x)\right)$. Find $g'(6)$.

(**c**) Let $y = h(x)$ be the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{y - f'(x)}{x}$ with $h(5) = 6$. Use Euler's method, starting at $x = 5$ with two steps of equal size, to approximate $h(3)$.

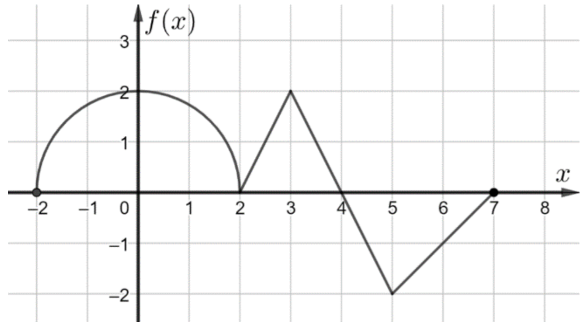


2. The graph of f', the derivative of the differentiable function f, consists of five linear segments and a semi circle and is shown in the figure above. The functions f and f' are defined on the closed interval $[-5, 10]$. It is known that $f(6) = -1$.

(**d**) For $0 \leq t \leq 10$ seconds, a particle moves along a horizontal axis with velocity v, measured in meters per second where $v(t) = f'(t)$ and t is measured in seconds. Evaluate $\int_0^{10} |v(t)|dt$. Using correct units, explain the meaning of this value in the context of the problem.

(**e**) Let $K(x) = 2x - \int_6^x f(t)dt$. Write the third degree Taylor polynomial to K about $x = 6$.

5 for 5: Calculus BC Day 1



The function f is continuous on the interval $[-2, 7]$ and consists of three line segments and a semi circle as shown in the figure above. The function g is defined by $g(x) = \int_{-2}^{x^2} f(t)dt$.

BC1: Let $h(x) = f(5x - 9)$. Find $h'(3)$.

BC2: Evaluate $\int_{-1}^{0} [f'(3 - 2x) - 4]\, dx$.

BC3: Write the 2nd degree Taylor polynomial for g about $x = 2$.

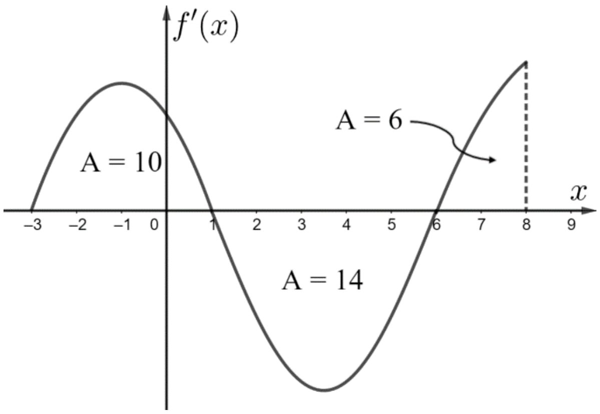
t seconds	0	1	4	6
$P(t)$ people per second	8	3	5	10

For $0 \leq t \leq 6$ seconds, people enter a school at the rate $P(t)$, measured in people per second.

BC4: Approximate $P'(5)$. Using correct units, interpret the meaning of $P'(5)$ in the context of the problem.

BC5: Use a left Riemann sum with the three subintervals indicated by the table above to approximate $\frac{1}{6}\int_{0}^{6} P(t)dt$. Using correct units, interpret the meaning of $\frac{1}{6}\int_{0}^{6} P(t)dt$ in the context of the problem.

5 for 5: Calculus BC Day 2



x	1	4	6	9
$g(x)$	3	1	0	-1
$g'(x)$	2	0	1	3

A portion of the graph of f', the derivative of the twice differentiable function f, is shown in the figure above. The areas of the regions bounded by the graph of f' and the x axis are labeled. It is known that $f(1) = -2$.

The function g is twice differentiable. Selected values of g and g' are shown in the table above.

BC1: Find all values of x in the open interval $-3 < x < 8$ for which the graph of f has horizontal tangent line. For each value of x, determine whether f has a relateive minimum, relative maximum, or neither a minimum nor a maximum at the x value. Justify your answers.

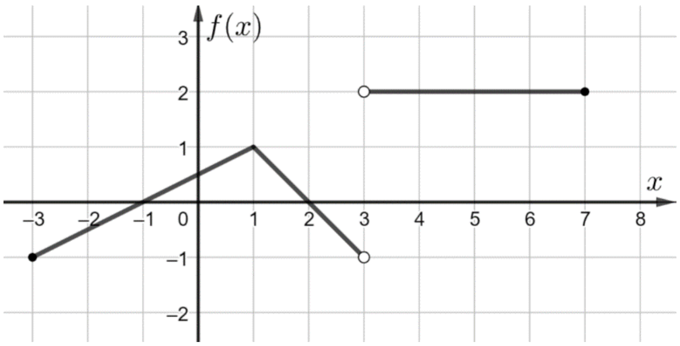
BC2: Find the minimum value of f on the closed interval $[-3, 8]$. Justify your answer..

BC3: Let $h(x) = \dfrac{e^{g(x)}}{3x}$. Find $h'(6)$.

BC4: For $t \geq 0$, a particle moves along a straight path with velocity $v(t) = f'(t)$. Find the total distance traveled by the particle from $t = 1$ to $t = 8$.

BC5: Evaluate $\int_1^9 x g''(x) dx$.

5 for 5: Calculus BC Day 3



x	0	1	3	5
$g(x)$	2	0	5	-1
$g'(x)$	7	4	-2	3

The function f is defined and continuous for all $x \geq -3$ except at $x = 3$. A portion of the graph of f, consisting of three linear pieces is shown in above. For $x \geq 7, f(x) = \dfrac{a}{x^{3p+2}}$ where a and p are constants.

The function g is differentiable for all values of x. Selected values of g and g', the derivative of g, are given in the table above.

BC1: Write an equation of the line tangent to g at $x = 3$. Use this tangent line to approximate $g(2)$.

BC2: Evaluate $\displaystyle\lim_{x \to -1} \frac{\int_{-3}^{x^2} f(t)dt}{x^3 + 1}$

BC3: Let $p(x) = \begin{cases} f(x)g'(x) & x < 3 \\ 4f'(x-3) & x \geq 3 \end{cases}$. *Is* $p(x)$ continuous at $x = 3$? Why or why not?

BC4: For $x \geq 7, f(x) > 0$. Let R be the region bounded by the graphs of $f(x)$, the horizontal line $y = -3$, and the vertical lines $x = 7$ and $x = 12$. The region R is the base of a solid. For this solid, each cross section perpendicular to the x axis is a square. Write, but do not evaluate, an integral expression for the volume of the solid.

BC5: For $x \geq 7$, let $b_n = f(n)$. For what values of p does $\displaystyle\sum_{n=7}^{\infty} b_n$ converge?