Final Practice FRQ Problems

1a. $\int_{-1}^{2} f'(6-4x)\,dx = \left[\frac{f(6-4x)}{-4}\right]\Big|_{-1}^{2} = \frac{f(-2)}{-4} - \frac{f(10)}{-4} = 0-0=0$

or u-sub: $u=6-4x$ $x=2 \rightarrow u=-2$ $\int_{10}^{-2} f'(u)\cdot\left(-\frac{1}{4}\right)du = -\frac{1}{4}f(u)\Big|_{10}^{-2}$

$\frac{du}{dx}=-4$ $x=-1 \rightarrow u=10$ $= -\frac{1}{4}f(-2)+\frac{1}{4}f(10)=0$

$-\frac{1}{4}du=dx$

1b. $\lim_{x\to 0} \frac{\int_{-4}^{2x} f(t)\,dt}{3x^2+x} \Rightarrow \frac{\int_{-4}^{0} f(t)\,dt}{0} \Rightarrow \frac{0}{0}$ L'Hopital's Rule

deriv of integral

$\lim_{x\to 0} \frac{\int_{-4}^{2x} f(t)\,dt}{3x^2+x} \overset{H}{=} \lim_{x\to 0} \frac{f(2x)\cdot 2}{6x+1} = \frac{f(0)\cdot 2}{1} = 8$

1c. g has a relative minimum when $g'(x)=f(x)$ changes from negative to positive. This occurs when $x=-2$

1d. $Q(-2)=4-g(2)= 4-\int_{-2}^{-2} f(t)\,dt = 4-0=+4$

$Q'(-2)=2(-2)-g'(-2)=-4-f(-2)=-4-0=-4$

$Q''(-2)=2-g''(-2)=2-f'(-2)=2-2=0$

$T_2(x) = 4-4(x+2)+\frac{0(x+2)^2}{2!} = 4-4(x+2)$

2a. $\int_{-2}^{2} \frac{1}{2}x\,g''(x)\,dx$ $u=\frac{1}{2}x$ $dv=g''(x)\,dx$

$du=\frac{1}{2}dx$ $v=g'(x)$

$= \frac{1}{2}x\,g'(x)\Big|_{-2}^{2} - \int_{-2}^{2}\frac{1}{2}g'(x)\,dx = \left[g'(2)-(-g'(-2))\right] - \left[\frac{1}{2}g(x)\right]\Big|_{-2}^{2}$

$= -2+6-\left[\frac{1}{2}g(2)-\frac{1}{2}g(-2)\right]$

$= 4-\left[\frac{1}{2}\cdot 4-\frac{1}{2}(-3)\right]$

2b. $\int_{-1}^{1} f'(1-2x)\,dx = \left[\frac{f(1-2x)}{-2}\right]\Big|_{-1}^{1} = \left[\frac{f(-1)}{-2} - \left(\frac{f(3)}{-2}\right)\right] = \frac{-1}{-2} - \left(\frac{2}{-2}\right) = \frac{1}{2}+1$

OR u-sub: $u=1-2x$ $x=1 \rightarrow u=-1$ $\int_{3}^{-1} f'(u)\left(-\frac{1}{2}\right)du = \left[-\frac{1}{2}f(u)\right]\Big|_{3}^{-1}$

$\frac{du}{dx}=-2$ $x=-1 \rightarrow u=3$

$-\frac{1}{2}du=dx$

$= -\frac{1}{2}f(-1)+\frac{1}{2}f(3)$

$= -\frac{1}{2}(-1)+\frac{1}{2}\cdot 2 = \frac{1}{2}+1$

2c $K(x) = \int_1^{\cos(x)} 2g(x)\,dx$

$K'(x) = 2g(\cos(x)) \cdot (-\sin(x))$

$K'(\frac{\pi}{2}) = 2g(\cos(\frac{\pi}{2})) \cdot (-\sin(\pi/2)) = 2[g(0)] \cdot (-\sin(\frac{\pi}{2})) = 2 \cdot 1 \cdot (-1) = -2$

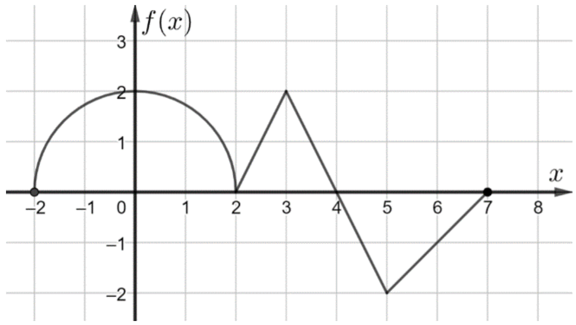
2d. $\sum_{n=1}^{\infty} \frac{6}{n^{\sqrt{2}p-1}} = 6\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}p-1}}$ p-series. p-series converge if the exponent is > 1

$\sqrt{2}p - 1 > 1$

$\sqrt{2}p > 2$

$p > \frac{2}{\sqrt{2}}$

5 for 5: Calculus AB Day 1 Solutions



The function f is continuous on the interval $[-2, 7]$ and consists of three line segments and a semi circle as shown in the figure above. The function g is defined by $g(x) = \int_{-2}^{x^2} f(t)dt$.

AB1: Find $g(2), g'(2),$ and $g''(2)$.

$$g(2) = \int_{-2}^{4} f(t)dt = \left[\frac{1}{2}\pi(2)^2 + \frac{1}{2}(2)(2)\right] = 2\pi + 2$$

□ $g'(x) = f(x^2)(2x) \Rightarrow g'(2) = f(4)(4) = (0)(4) = 0$

$$g''(x) = f(x^2)(2) + f'(x^2)(2x)^2 \Rightarrow g''(2) = f(4)(2) + f'(4)(4)^2 = 16f'(4) = 16(-2) = -32$$

AB2: Let $h(x) = f(5x - 9)$. Find $h'(3)$.

$$h'(x) = f'(5x-9)(5) \Rightarrow h'(3) = f'(5(3)-9)(5) = 5f'(6) = 5(1) = 5$$

AB3: Evaluate $\int_{-1}^{0} [f'(3 - 2x) - 4]\, dx$.

$$\int_{-1}^{0}\left[f'(3-2x)-4\right]\,dx = -\frac{1}{2}\int_{-1}^{0}\left[f'\left(\underbrace{3-2x}_{u}\right)\right](-2dx) - \int_{-1}^{0}[4]\,dx$$

$$= -\frac{1}{2}\left[f(3-2x)\right]_{-1}^{0} - \left[4x\right]_{-1}^{0} = -\frac{1}{2}\left[f(3)-f(5)\right] - \left[-4(-1)\right] = -\frac{1}{2}\left[(2)-(-2)\right] - [4] = -6$$

t seconds	0	1	4	6
$P(t)$ people per second	8	3	5	10

For $0 \leq t \leq 6$ seconds, people enter a school at the rate $P(t)$, measured in people per second.

AB4: Approximate $P'(5)$. Using correct units, interpret the meaning of $P'(5)$ in the context of the problem.

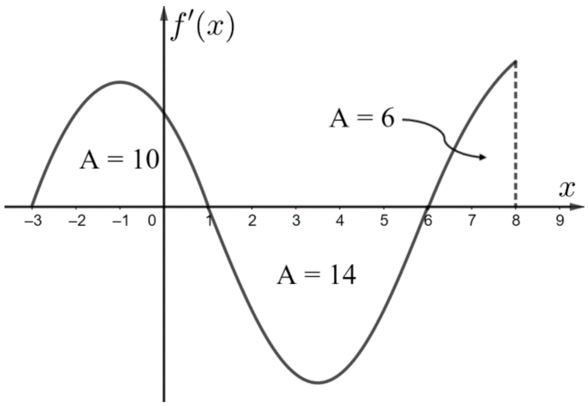
$$P'(5) \approx \frac{P(6) - P(4)}{6 - 4} = \frac{(10) - (5)}{6 - 4} = \frac{5}{2}$$

The rate people enter a school is changing at a rate of $P'(5)$ people per second per second at $t = 5$ seconds.

AB5: Use a left Riemann sum with the three subintervals indicated by the table above to approximate $\int_0^6 P(t)dt$.

$$\int_0^6 P(t)dt \approx P(0)(1) + P(1)(3) + P(4)(2) = (8)(1) + (3)(3) + (5)(2) = 27$$

5 for 5: Calculus AB Day 2 Solutions



x	1	4	6	9
$g(x)$	3	1	0	-1
$g'(x)$	2	0	1	3

A portion of the graph of f', the derivative of the twice differentiable function f, is shown in the figure above. The areas of the regions bounded by the graph of f' and the x axis are labeled. It is known that $f(1) = -2$.

The function g is twice differentiable. Selected values of g and g' are shown in the table above.

AB1: Find all values of x in the open interval $-3 < x < 8$ for which the graph of f has horizontal tangent line. For each value of x, determine whether f has a relateive minimum, relative maximum, or neither a minimum nor a maximum at the x value. Justify your answers.

horizontal tangent line $\Rightarrow f'(x) = 0 \Rightarrow x = 1, 6$

At $x = 1$ there is a relative maximum because $f'(x)$ changes from positive to negative.

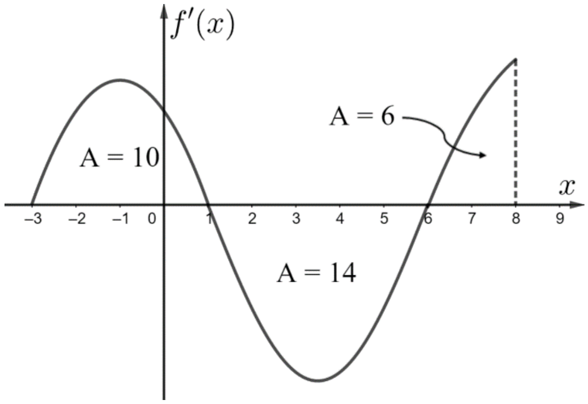
At $x = 6$ there is a relative minimum because $f'(x)$ changes from negative to positive.

AB2: Find the minimum value of f on the closed interval $[-3, 8]$. Justify your answer.

relative minimum candidates: $x = 6$ endpoints: $x = -3, 8$

x	$f(x)$
-3	$-2 - \int_{-3}^{1} f'(x)\,dx = -2 - (10) = -12$
6	$-2 + \int_{1}^{6} f'(x)\,dx = -2 - (14) = -16$
8	$-2 + \int_{1}^{8} f'(x)\,dx = -2 - (14) + 6 = -10$

5 for 5: Calculus AB Day 2



x	1	4	6	9
$g(x)$	3	1	0	−1
$g'(x)$	2	0	1	3

A portion of the graph of f', the derivative of the twice differentiable function f, is shown in the figure above. The areas of the regions bounded by the graph of f' and the x axis are labeled. It is known that $f(1) = -2$.

The function g is twice differentiable. Selected values of g and g' are shown in the table above.

AB3: Let $h(x) = \dfrac{e^{g(x)}}{3x}$. Find $h'(6)$.

$$h'(x) = \frac{(3x)\left(e^{g(x)}\right)(g'(x)) - (3)\left(e^{g(x)}\right)}{(3x)^2}$$

$$h'(6) = \frac{(18)\left(e^{g(6)}\right)(g'(6)) - (3)\left(e^{g(6)}\right)}{(18)^2} = \frac{(18)\left(e^{0}\right)(1) - (3)\left(e^{0}\right)}{(18)^2} = \frac{(18) - (3)}{(18)^2} = \frac{15}{(18)^2} = \frac{5}{108}$$

AB4: Is there a time $c, 1 < c < 9$, such that $g'(c) = -\dfrac{1}{2}$? Give a reason for your answer.

$$\frac{g(9) - g(1)}{9 - 1} = \frac{-1 - 3}{8} = \frac{-4}{8} = -\frac{1}{2}$$

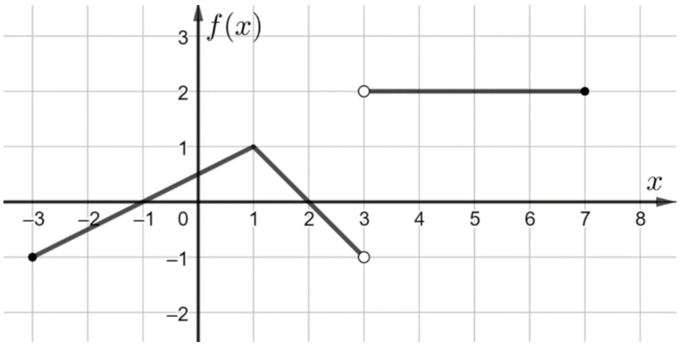
g is differentiable and therefore continuous for all x on the interval $1 < x < 9$, so the Mean Value Theorem guarantees there is at least one number c between 1 and 9 such that

$$g'(c) = \frac{g(9) - g(1)}{9 - 1} = -\frac{1}{2}.$$

AB5: Evaluate $\displaystyle\int_1^4 [g(x)]^2 g'(x)\,dx$.

$$\int_1^4 \underbrace{\left[g(x)\right]}_{u}^2 \underbrace{g'(x)\,dx}_{du} = \int_{g(1)}^{g(4)} u^2\,du = \left[\frac{1}{3}u^3\right]_3^1 = \frac{1}{3}\left[(1)^3 - (3)^3\right] = \frac{1}{3}[1 - 27] = -\frac{26}{3}$$

5 for 5: Calculus AB Day 3 Solutions



x	0	1	3	5
$g(x)$	2	0	5	-1
$g'(x)$	7	4	-2	3

The function f is defined and continuous for all $x \geq -3$ except at $x = 3$. A portion of the graph of f, consisting of three linear pieces is shown in the figure above.

The function g is differentiable for all values of x. Selected values of g and g', the derivative of g, are given in the table above.

AB1: Write an equation of the line tangent to g at $x = 3$. Use this tangent line to approximate $g(2)$.

$T(x) = g(3) + g'(3)(x-3) = 5 - 2(x-3)$

$g(2) \approx T(2) = 5 - 2(2-3) = 5 - 2(-1) = 7$

AB2: Evaluate $\displaystyle\lim_{x\to-1} \frac{\int_{-3}^{x^2} f(t)dt}{x^3+1}$

$$\lim_{x\to-1} \int_{-3}^{x^2} f(t)\,dt = \lim_{x\to-1} \int_{-3}^{1} f(t)\,dt = \left[-\frac{1}{2}(2)(1) + \frac{1}{2}(2)(1)\right] = [-1+1] = 0$$

$$\lim_{x\to-1}(x^3+1) = -1+1 = 0$$

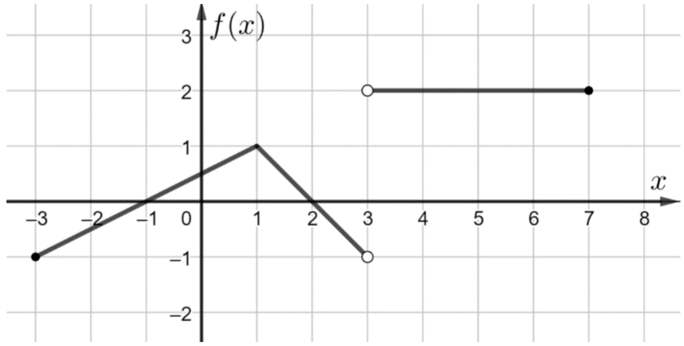
$\displaystyle\lim_{x\to-1} \frac{\int_{-3}^{x^2} f(t)\,dt}{x^3+1}$ produces the indeterminant form $\frac{0}{0}$ so we can apply l'Hospital's Rule

$$\lim_{x\to-1} \frac{\int_{-3}^{x^2} f(t)\,dt}{x^3+1} = \underbrace{\lim_{x\to-1} \frac{f(x^2)(2x)}{3x^2}}_{\text{l'Hospital's Rule}} = \frac{f\left((-1)^2\right)(2(-1))}{3(-1)^2} = \frac{(1)(-2)}{3(1)} = \frac{-2}{3}$$

AB3: Let $k(x) = g(f(x))$. Find $k'(2)$.

$k'(x) = g'(f(x))(f'(x))$

$k'(2) = g'(f(2))(f'(2)) = g'(0)(-1) = (7)(-1) = -7$



x	0	1	3	5
$g(x)$	2	0	5	-1
$g'(x)$	7	4	-2	3

The function f is defined and continuous for all $x \geq -3$ except at $x = 3$. A portion of the graph of f, consisting of three linear pieces is shown in the figure above.

The function g is differentiable for all values of x. Selected values of g and g', the derivative of g, are given in the table above.

AB4: Let $p(x) = \begin{cases} f(x)g'(x) & x < 3 \\ 4f'(x-3) & x \geq 3 \end{cases}$. *Is* $p(x)$ continuous at $x = 3$? Why or why not?

$\lim\limits_{x\to 3^-}\left[f(x)g'(x)\right] = (-1)g'(3) = (-1)(-2) = 2$ $\lim\limits_{x\to 3^+}\left[4f'(x-3)\right] = 4f'(0) = (4)\left(\frac{1}{2}\right) = 2$

$p(3) = 4f'(0) = (4)\left(\frac{1}{2}\right) = 2$

$p(x)$ is continuous at $x = 3$ because $p(3) = \lim\limits_{x\to 3^-} p(x) = \lim\limits_{x\to 3^+} p(x)$.

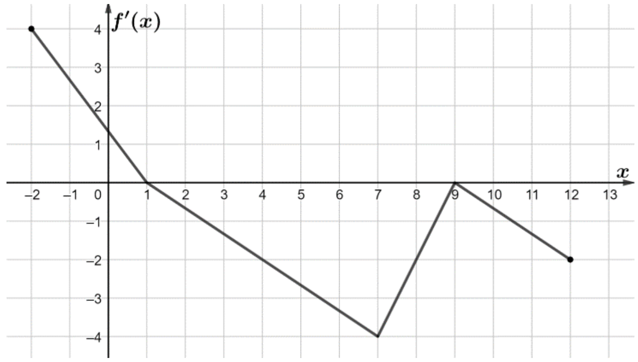
AB5: If $\int_{-3}^{10} f(x)dx = 5$, find the value of $\int_{7}^{10} f(x)dx$. Show the work that leads to your answer.

$$\int_{-3}^{10} f(x)dx = \int_{-3}^{7} f(x)dx + \int_{7}^{10} f(x)dx$$

$$5 = \left[-\frac{1}{2}(2)(1) + \frac{1}{2}(2)(1) + (4)(2)\right] + \int_{7}^{10} f(x)dx$$

$$5 = [8] + \int_{7}^{10} f(x)dx \Rightarrow \int_{7}^{10} f(x)dx = -3$$

5 for 5: Calculus AB Day 4 Solutions



The function f is differentiable on the interval $[-2, 12]$ and consists of three line segments as shown in the figure above. It is known that $f(4) = 14$

AB1: On what open intervals is the graph of f both decreasing and concave down? Give a reason for your answer.

f is decreasing $\Rightarrow f'(x) \le 0$ f is concave down $\Rightarrow f'(x)$ is decreasing

f is decreasing and concave down on the open intervals $(1,7)$ and $(9,12)$.

AB2: Let $g(x) = f(x)f'(x)$. Find $g'(4)$.

$$g'(x) = f'(x)f'(x) + f(x)f''(x)$$

$$g'(4) = f'(4)f'(4) + f(4)f''(4) = (-2)(-2) + (14)\left(-\frac{2}{3}\right) = 4 - \frac{28}{3}$$

AB3: Evaluate $\int_{-2}^{12} [3 - 2f'(x)]\, dx$.

$$\int_{-2}^{12}\left[3-2f'(x)\right]dx = \int_{-2}^{12}[3]dx - \int_{-2}^{12}\left[2f'(x)\right]dx = \left[3x\right]_{-2}^{12} - 2\int_{-2}^{12}\left[f'(x)\right]dx$$

$$= \left[36-(-6)\right] - 2\left[\frac{1}{2}(4)(3) - \frac{1}{2}(8)(4) - \frac{1}{2}(3)(2)\right]$$

$$= [42] - 2[6-26-3] = [42] - 2[-23] = [42] + [46] = 88$$

What they wrote is wrong
This is correct: 42-2[6-16-3]

68 is correct answer

t	0	0.2	0.4	0.5	0.6	0.8	1.0
$W(t)$	4	5.7	9.3	12.2	16.3	29.3	53.2

Consider the differential equation $\frac{dW}{dt} = 9 - W^2$. Let $y = W(t)$ be the particular solution to the differential equation with the initial condition $W(0) = 4$. The function W is twice differentiable with selected values of W given in the table above.

AB4: Find $\frac{d^2W}{dt^2}$ in terms of W.

$$\frac{d^2W}{dt^2} = \frac{d}{dt}\left(9 - W^2\right) = -2W\frac{dW}{dt} = -2W\left(9 - W^2\right)$$

AB5: Use a midpoint Riemann sum with the three subintervals indicated by the table above to approximate $\int_0^1 W(t)dt$.

$$\int_0^1 W(t)\,dt \approx \left[(0.4)W(0.2) + (0.2)W(0.5) + (0.4)W(0.8)\right] = \left[(0.4)(5.7) + (0.2)(12.2) + (0.4)(29.3)\right]$$

$$= \left[(2.28) + (2.44) + (11.72)\right] = 16.44$$