

Graph of $f$

1. The continuous function $f$ is defined on the closed interval $-4 \leq x \leq 12$ and consists only of line segments as shown in the figure above. The function $g$ is given by $g(x)=\int_{-2}^{x} f(t) d t$.
a. Evaluate $\int_{-1}^{2} f^{\prime}(6-4 x) d x$
b. Find $\lim _{x \rightarrow 0} \frac{\int_{-4}^{2 x} f(t) d t}{3 x^{2}+x}$
c. On the interval $-4 \leq x \leq 12$, identify the $x$-value(s) at which $g$ has a relative minimum. Justify your answer.
d. Let $Q(x)=x^{2}-g(x)$. Write the second degree Taylor Polynomial for Q about $\mathrm{x}=-2$.


| $x$ | -2 | -1 | 0 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $g(x)$ | -3 | -2 | 1 | 4 | 5 |
| $g^{\prime}(x)$ | 6 | 2 | -3 | -2 | 3 |

Graph of $f$
2. The function $f$ is defined and continuous on the closed interval $-4 \leq x \leq 4$ and is piecewiselinear as shown above. The function $g$ is twice-differentiable for all values of $x$. Selected values of $g$ and $g^{\prime}$, the derivative of $g$, are given in the table above. The function $h(x)=\frac{b}{x^{\sqrt{2} p-1}}$ is defined for $x>0$ where $b$ and $p$ are both constants.
a. $\int_{-2}^{2} \frac{1}{2} x g^{\prime \prime}(x) d x$
b. $\int_{-1}^{1} f^{\prime}(1-2 x) d x$
c. $\quad k(x)=\int_{1}^{\cos (x)} 2 g(x) d x$. What is the value of $k^{\prime}\left(\frac{\pi}{2}\right)$ ?
d. Let $a_{n}=h(n)$. For what values of $p$ does $\sum_{n=1}^{\infty} a_{n}$ converge?

5 for 5: Calculus AB Day 1


The function $f$ is continuous on the interval $[-2,7]$ and consists of three line segments and a semi circle as shown in the figure above. The function $g$ is defined by $g(x)=\int_{-2}^{x^{2}} f(t) d t$.

AB1: Find $g(2), g^{\prime}(2)$, and $g^{\prime \prime}(2)$.

AB2: Let $h(x)=f(5 x-9)$. Find $h^{\prime}(3)$.

AB3: Evaluate $\int_{-1}^{0}\left[f^{\prime}(3-2 x)-4\right] d x$.

| $t$ <br> seconds | 0 | 1 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $P(t)$ <br> people per second | 8 | 3 | 5 | 10 |

For $0 \leq t \leq 6$ seconds, people enter a school at the rate $P(t)$, measured in people per second.

AB4: Approximate $P^{\prime}(5)$. Using correct units, interpret the meaning of $P^{\prime}(5)$ in the context of the problem.

AB5: Use a left Riemann sum with the three subintervals indicated by the table above to approximate $\int_{0}^{6} P(t) d t$.

5 for 5: Calculus AB Day 2


| $x$ | 1 | 4 | 6 | 9 |
| :---: | :---: | :---: | :---: | :---: |
| $g(x)$ | 3 | 1 | 0 | -1 |
| $g^{\prime}(x)$ | 2 | 0 | 1 | 3 |

A portion of the graph of $f^{\prime}$, the derivative of the twice differentiable function $f$, is shown in the figure above. The areas of the regions bounded by the graph of $f^{\prime}$ and the $x$ axis are labeled. It is known that $f(1)=-2$.

The function $g$ is twice differentiable. Selected values of $g$ and $g^{\prime}$ are shown in the table above.

AB1: Find all values of $x$ in the open interval $-3<x<8$ for which the graph of $f$ has horizontal tangent line. For each value of $x$, determine whether $f$ has a relateive minimum, relative maximum, or neither a minimum nor a maximum at the $x$ value. Justify your answers.

AB2: Find the minimum value of $f$ on the closed interval $[-3,8]$. Justify your answer..

AB3: Let $h(x)=\frac{e^{g(x)}}{3 x}$. Find $h^{\prime}(6)$.

AB4: Is there a time $c, 1<c<9$, such that $g^{\prime}(c)=-\frac{1}{2}$ ? Give a reason for your answer.

AB5: Evaluate $\int_{1}^{4}[g(x)]^{2} g^{\prime}(x) d x$.


| $x$ | 0 | 1 | 3 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| $g(x)$ | 2 | 0 | 5 | -1 |
| $g^{\prime}(x)$ | 7 | 4 | -2 | 3 |

The function $f$ is defined and continuous for all $x \geq-3$ except at $x=3$. A portion of the graph of $f$, consisting of three linear pieces is shown in the figure above.

The function $g$ is differentiable for all values of $x$. Selected values of $g$ and $g^{\prime}$, the derivative of $g$, are given in the table above.

AB1: Write an equation of the line tangent to $g$ at $x=3$. Use this tangent line to approximate $g(2)$.

AB2: Evaluate $\lim _{x \rightarrow-1} \frac{\int_{-3}^{x^{2}} f(t) d t}{x^{3}+1}$

AB3: Let $k(x)=g(f(x))$. Find $k^{\prime}(2)$.

AB4: Let $p(x)=\left\{\begin{array}{ll}f(x) g^{\prime}(x) & x<3 \\ 4 f^{\prime}(x-3) & x \geq 3\end{array}\right.$. Is $p(x)$ continuous at $x=3$ ? Why or why not?

AB5: If $\int_{-3}^{10} f(x) d x=5$, find the value of $\int_{7}^{10} f(x) d x$. Show the work that leads to your answer.

5 for 5: Calculus AB Day 4


The function $f$ is differentiable on the interval $[-2,12]$ and consists of three line segments as shown in the figure above. It is known that $f(4)=14$
AB1: On what open intervals is the graph of $f$ both decreasing and concave down? Give a reason for your answer.

AB2: Let $g(x)=f(x) f^{\prime}(x)$. Find $g^{\prime}(4)$.

AB3: Evaluate $\int_{-2}^{12}\left[3-2 f^{\prime}(x)\right] d x$.

| $t$ | 0 | 0.2 | 0.4 | 0.5 | 0.6 | 0.8 | 1.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $W(t)$ | 4 | 5.7 | 9.3 | 12.2 | 16.3 | 29.3 | 53.2 |

Consider the differential equation $\frac{d W}{d t}=9-W^{2}$. Let $y=W(t)$ be the particular solution to the differential equation with the initial condition $W(0)=4$. The function $W$ is twice differentiable with selected values of $W$ given in the table above.
AB4: Find $\frac{d^{2} W}{d t^{2}}$ in terms of $W$.

AB5: Use a midpoint Riemann sum with the three subintervals indicated by the table above to approximate $\int_{0}^{1} W(t) d t$.

