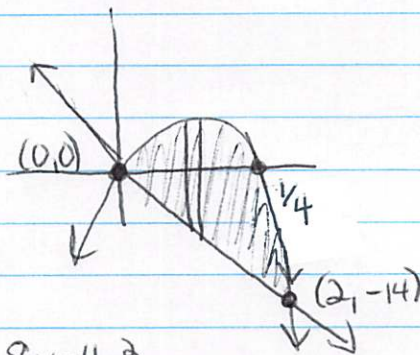


Integrals Applied to Area + Volume MC HUK:

① $y = x - 4x^2, y = -7x$

$0 = x(1-4x)$
at $x=0, x=1/4$

$x - 4x^2 = -7x$
 $0 = 4x^2 - 8x$
 $0 = 4x(x-2) = 0$ at
 $x=0, x=2$

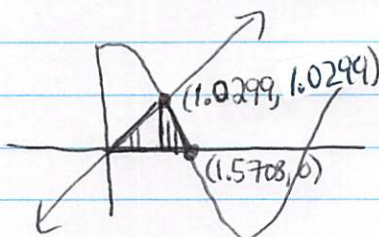


$A = l \cdot w: l = y_T - y_B = x - 4x^2 - (-7x) = 8x - 4x^2$

$A = \int_0^{1/4} (8x - 4x^2) dx = \left[\frac{8x^2}{2} - \frac{4x^3}{3} \right]_0^{1/4} = \left[4 \cdot \frac{1}{4} - \frac{4}{3} \left(\frac{1}{64} \right) \right] - 0 = 1 - \frac{1}{48} = \frac{48}{48} - \frac{1}{48} = \frac{47}{48}$

② $y = 2\cos x, y = x, x\text{-axis}$

$A = \int_0^{1.0299} (x - 0) dx + \int_{1.0299}^{1.5708} (2\cos x - 0) dx =$
 $0.53035 + 0.2855 = 0.81585$ A



* found area of 2 regions + added them together.

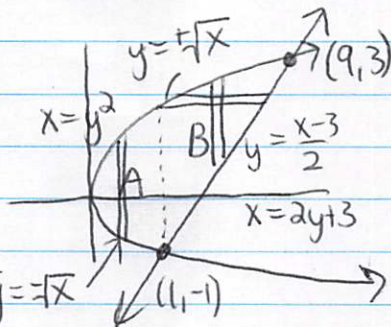
③ $\int_{-4}^1 [f(x) + 2] dx = \int_{-4}^1 f(x) dx + \int_{-4}^1 2 dx = -5 + 2 - 1 + 2(1 - (-4)) = -4 + 10 = 6$

④ $A = l \cdot w \Rightarrow l = y_T - y_B = 5 - (1+x^2) = 4 - x^2$ $A = \int_1^2 (4 - x^2) dx$ B
 $5 = 1 + x^2 \quad 4 = x^2 \quad x = \pm 2$

⑤ II $A \quad y_T - y_B = 2\sqrt{x}$

B. $y_T - y_B = \sqrt{x} - \left(\frac{x-3}{2}\right)$

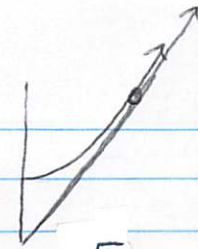
$A = \int_0^1 2\sqrt{x} dx + \int_1^9 \left[\sqrt{x} - \left(\frac{x-3}{2}\right) \right] dx$ $y = \sqrt{x}$



IV $x_R - x_L = 2y + 3 - y^2$

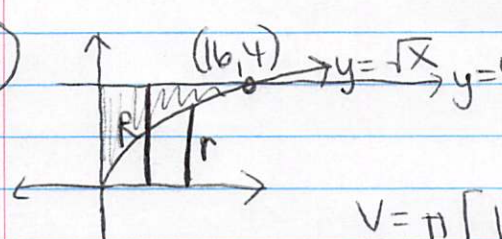
$A = \int_{-1}^3 (2y + 3 - y^2) dy$

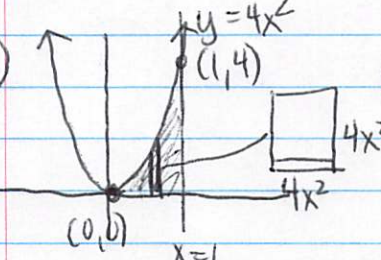
C

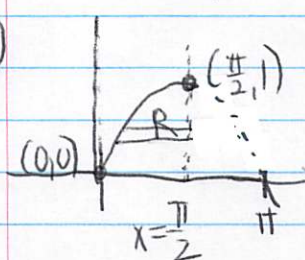
(6)  tangent when $y' = y'$ $y' = k = 2x = y'$ (plus 2x in fact)

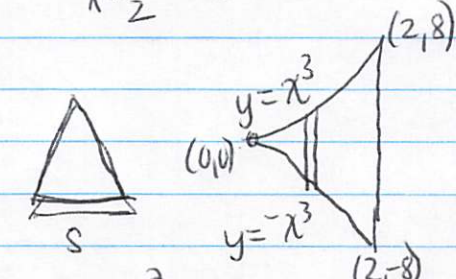
(B) $y = 2x \cdot x - 10 = 2x^2 - 10 = x^2 - 4$
 $x^2 = 6$
 $x = \sqrt{6}$
 $k = 2x = 2\sqrt{6}$

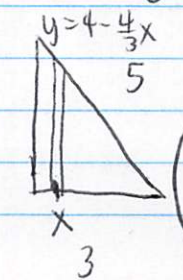
$A = \int_0^{\sqrt{6}} [(x^2 - 4) - (2\sqrt{6}x - 10)] dx = 4.899$

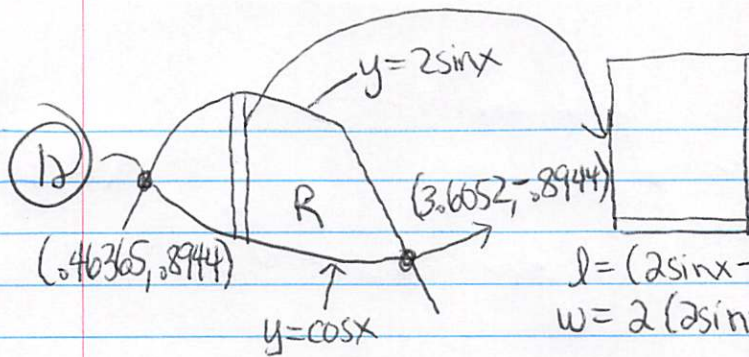
(7)  $y = \sqrt{x}$ $y = 4$ $R = 4$ $V = \pi \int_0^{16} (4^2 - (\sqrt{x})^2) dx = \pi \int_0^{16} (16 - x) dx$
 $r = \sqrt{x}$ $V = \pi [16x - \frac{x^2}{2}]_0^{16} = \pi [(16^2 - \frac{16^2}{2}) - 0] = \pi \frac{16^2}{2} = 128\pi$ (C)

(8)  $y = 4x^2$ $A_s = 16x^4$ $V_s = 16x^4 dx$ $V_{solid} = 16 \int_0^1 x^4 dx = 16 \frac{x^5}{5} \Big|_0^1 = \frac{16}{5} = \frac{16}{5}$ (D)

(9)  $y = \sin x$ $A = \pi r^2 = \pi (\frac{\pi}{2} - \sin^{-1}(y))^2$
 $\sin^{-1}(y) = x$ $r = x_R - x_L = \frac{\pi}{2} - \sin^{-1}(y)$ $V = \pi \int_0^1 (\frac{\pi}{2} - \sin^{-1}(y))^2 dy = 3.586$

(10)  $y = x^3$ $S = y_r - y_b = x^3 - (-x^3) = 2x^3$
 $y = -x^3$ $A = \frac{\sqrt{3}}{4} S^2 = \frac{\sqrt{3}}{4} (2x^3)^2 = \sqrt{3} x^6$
 $V = \sqrt{3} \int_0^2 x^6 dx = \sqrt{3} [\frac{x^7}{7}]_0^2 = \frac{\sqrt{3}}{7} \cdot (128) = \frac{128\sqrt{3}}{7}$ (E)

(11)  $y = 4 - \frac{4}{3}x$ $V = (4 - \frac{4}{3}x) dx \cdot f(x)$
 $V = \int_0^3 (4 - \frac{4}{3}x) \cdot f(x) dx$ (A)



$$A = l \cdot w = 2(2 \sin x - \cos x)^2$$

$$V = 2 \int_{0.46365}^{3.652} (2 \sin x - \cos x)^2 dx$$

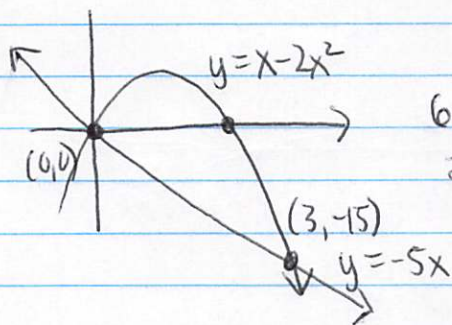
$$V = 15.708 \quad \Delta$$

13

$$0 = x - 2x^2$$

$$0 = x(1 - 2x)$$

$$x = 0, x = 1/2$$



$$x - 2x^2 = -5x$$

$$6x - 2x^2 = 0$$

$$2x(3 - x) = 0$$

$$x = 0, x = 3$$

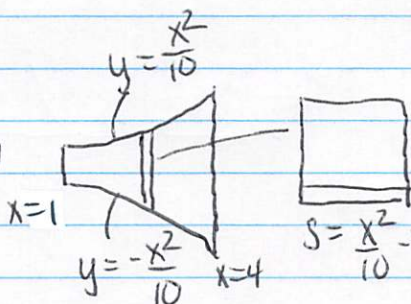
$$A = \int_0^3 (x - 2x^2) - (-5x) dx =$$

$$A = \int_0^3 (6x - 2x^2) dx =$$

$$\left[\frac{6x^2}{2} - \frac{2x^3}{3} \right]_0^3 = \left[3(9) - \frac{2(27)}{3} \right] - 0 =$$

$$27 - 18 = 9 \quad \Delta$$

14

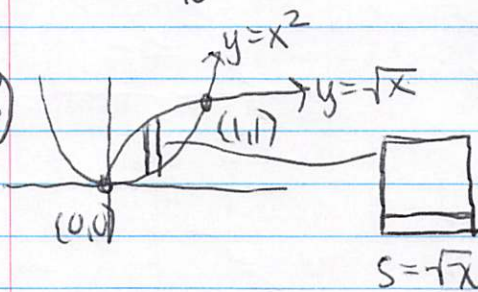


$$s = \frac{x^2}{10} - \left(-\frac{x^2}{10}\right) = \frac{2x^2}{10} = \frac{x^2}{5}$$

$$A = s^2 = \left(\frac{x^2}{5}\right)^2 = \frac{x^4}{25}$$

$$V = \int_1^4 \frac{x^4}{25} dx = 8.184 \quad \Delta$$

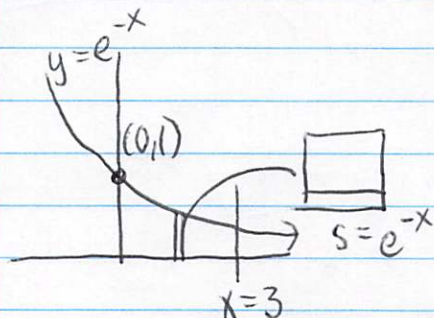
15



$$A = s^2 = (\sqrt{x} - x^2)^2$$

$$V = \int_0^1 (\sqrt{x} - x^2)^2 dx = 0.12857 \quad \Delta$$

16



$$A = s^2 = (e^{-x})^2 = e^{-2x}$$

$$V = \int_0^3 e^{-2x} dx = \left[-\frac{1}{2} e^{-2x} \right]_0^3 = -\frac{1}{2} [e^{-6} - e^0]$$

$$= -\frac{1}{2} [e^{-6} - 1]$$

$$A = \frac{1}{2} [1 - e^{-6}] \quad \Delta$$

17

$A = \pi a^2 = \pi (x^2)^2 = \pi x^4$
 $V = \pi \int_0^2 \pi x^4 dx =$
 $V = \pi \left[\frac{x^5}{5} \right]_0^2 = \pi \left(\frac{32}{5} \right) E$

18

$A = \pi (R^2 - r^2) = \pi (2^2 - \sqrt{y}^2) = \pi (4 - y)$
 $R = 2 - 0 = 2$
 $r = x_R - x_L$
 $x = 2$
 $r = \sqrt{y} - 0$
 $V = \pi \int_0^4 (4 - y) dy = \pi \left[4y - \frac{y^2}{2} \right]_0^4$
 $V = \pi \left[\left(16 - \frac{16}{2} \right) - 0 \right] = \pi [16 - 8] = 8\pi$

19

$A = \pi r^2 = \pi (\sqrt{y})^2 = \pi y$
 $r = \sqrt{y}$
 $V = \pi \int_0^4 y dy = \pi \left[\frac{y^2}{2} \right]_0^4 = \pi \left[\frac{16}{2} - 0 \right] = 8\pi$

20

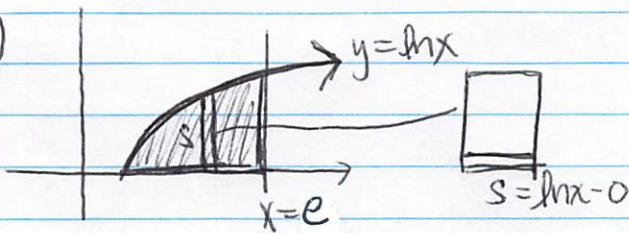
$A = \pi [R^2 - r^2] = \pi [4^2 - (x^2)^2] = \pi [16 - x^4]$
 $R = 4$
 $r = x^2$
 $y_T = y_B$
 $V = \pi \int_{-2}^2 (16 - x^4) dx = 2\pi \int_0^2 (16 - x^4) dx =$
 $V = 2\pi \left[16x - \frac{x^5}{5} \right]_0^2 = 2\pi \left[32 - \frac{32}{5} \right] = 2\pi \left[\frac{4}{5} (32) \right] = \frac{256\pi}{5} C$

21

$A = \pi r^2 = \pi (4 - x^2)^2 = \pi (16 - 8x^2 + x^4)$
 $r = y_T - y_B = 4 - x^2$
 $V = \pi \int_{-2}^2 (16 - 8x^2 + x^4) dx = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 =$
 $2\pi \left[32 - \frac{64}{3} + \frac{32}{5} \right] - 0 = 2\pi \left[\frac{480}{15} - \frac{320}{15} + \frac{96}{15} \right] = 2\pi \left[\frac{256}{15} \right] = \frac{512\pi}{15}$

D

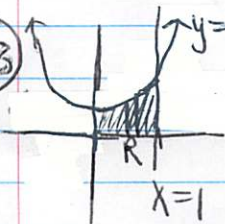
22



$$A = s^2 = (\ln x)^2$$

$$V = \int_1^e (\ln x)^2 dx = \boxed{1/C}$$

23

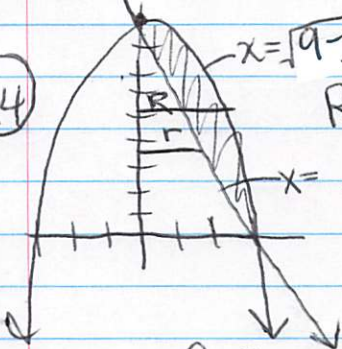


$$R = y_1 - y_2 = (x^2 + 1)^3 - 0 = (x^2 + 1)^3$$

$$A = \pi r^2 = \pi [(x^2 + 1)^3]^2 = \boxed{\pi(x^2 + 1)^6} \quad \text{D}$$

$$V = \pi \int_0^1 (x^2 + 1)^6 dx$$

24



$$R = \sqrt{9-y} \quad r = -\frac{1}{3}(y-9) \quad A = \pi(R^2 - r^2)$$

$$V = \pi \int_0^9 [R^2 - r^2] dy$$

$$V = \pi \int_0^9 (\sqrt{9-y})^2 - (\frac{1}{3}(9-y))^2 dy$$

$$V = 13.5\pi \quad \text{C}$$

$$\begin{aligned} y &= 9 - 3x & y &= 9 - x^2 \\ y - 9 &= -3x & x^2 &= 9 - y \\ -\frac{1}{3}(y-9) &= x & x &= \sqrt{9-y} \\ \frac{1}{3}(9-y) &= x & & \end{aligned}$$

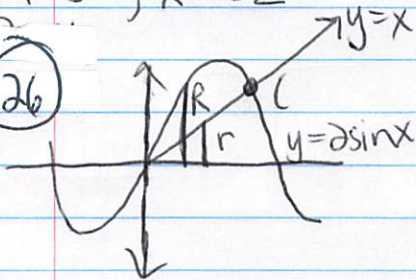
25

$$A = \int_0^2 (5 - (x^2 + 1)) dx = \int_0^2 (4 - x^2) dx = 4x - \frac{x^3}{3} \Big|_0^2 = 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3} = \frac{16}{3}$$

B

$$x^2 + 1 = 5, \quad x = \pm 2$$

26

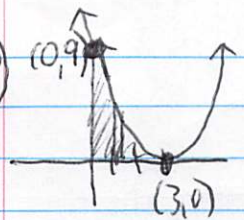


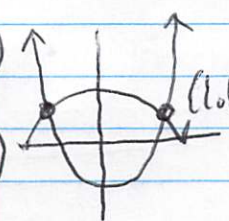
$$A = \pi(R^2 - r^2) = \pi((2 \sin x)^2 - x^2)$$

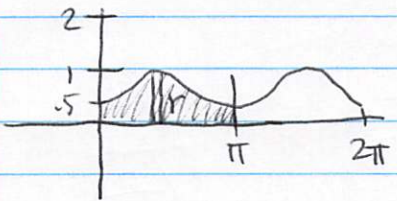
$$R = 2 \sin x$$

$$r = x$$

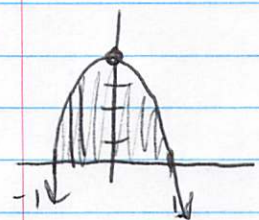
$$V = \pi \int_0^{1.0895} [(2 \sin x)^2 - x^2] dx = 2.126\pi = \boxed{6.678} \quad \text{D}$$

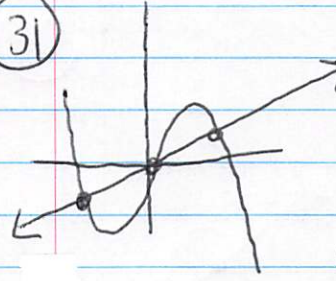
27)  $A = \pi r^2 = \pi [(x-3)^2]^2 = \pi (x-3)^4$
 $r = (x-3)^2$ B $V = \pi \int_0^3 (x-3)^4 dx$

28)  $A = \int_{-1.1373}^{1.1373} [\sqrt{4-x^2} - (e^{x^2}-2)] dx = 5.050$
 (use y-vals!) D

29)  $A = \pi r^2 = \pi (\cos(\cos x))^2$
 $r = \cos(\cos x)$ C $V = \pi \int_0^\pi [\cos(\cos x)]^2 dx = 1.922\pi = 6.0397$

30) $0 = 4(1-x^2) = 4(1-x)(1+x) = 0$
 at $x = 1, -1$

 $A = \int_{-1}^1 (4-4x^2) dx = 2 \int_0^1 (4-4x^2) dx = 2 [4x - \frac{4x^3}{3}]_0^1 = 2 [(4 - \frac{4}{3}) - 0] = 2 [\frac{12}{3} - \frac{4}{3}] = 2 (\frac{8}{3}) = \frac{16}{3}$ B

31)  $5x - x^3 = x$
 $4x - x^3 = 0$
 $x(4-x^2) = 0$
 at $x = 0, x = \pm 2$

$\int_{-2}^0 [x - (5x - x^3)] dx + \int_0^2 [(5x - x^3) - x] dx$
 $\int_{-2}^0 (-4x + x^3) dx + \int_0^2 (4x - x^3) dx$
 $= 2 \int_0^2 (4x - x^3) dx$ D

