1.3 Evaluating Limits at a Point from Graphs, Tables, and Functions

DO NOW:

$\lim_{x\to 3}(-9+x+x^2)$

$= 3$

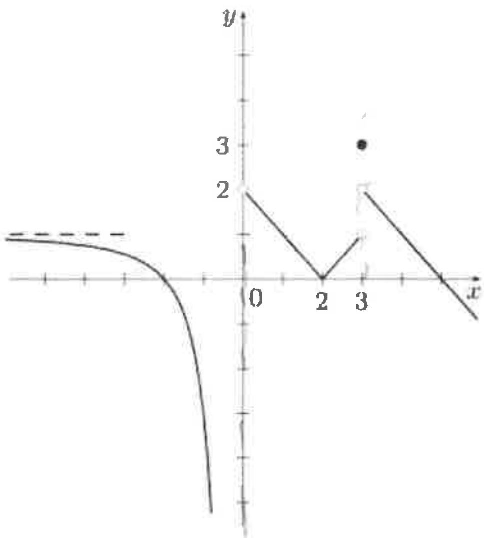
$\lim_{x\to -4}\frac{x+4}{x^2+6x+8}$

$\lim_{x\to -4}\frac{\cancel{(x+4)}}{\cancel{(x+4)}(x+2)}$

$= \frac{-1}{2}$

Intensive Practice

1. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$ DNE

(b) $f(2) =$ 0

(c) $f(3) =$ 3

(d) $\lim_{x\to 0^-} f(x) =$ $-\infty$

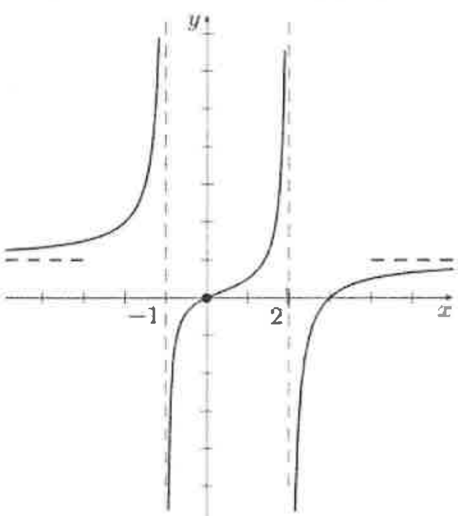
(e) $\lim_{x\to 0} f(x) =$ DNE

(f) $\lim_{x\to 3^+} f(x) =$ 2

(g) $\lim_{x\to 3} f(x) =$ DNE

(h) $\lim_{x\to -\infty} f(x) =$ 1

2. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$ 0

(b) $f(2) =$ DNE

(c) $f(3) =$ 0

(d) $\lim_{x\to -1} f(x) =$ DNE

(e) $\lim_{x\to 0} f(x) =$ 0

(f) $\lim_{x\to 2} f(x) =$ $-\infty$

(g) $\lim_{x\to \infty} f(x) =$ 1

Answer the following questions for the piecewise defined function $f(x)$ described on the right hand side.

$$f(x) = \begin{cases} \sin(\pi x) & \text{for } x < 1, \\ 2^{x^2} & \text{for } x > 1. \end{cases}$$

(a) $f(1) =$ DNE b/c neither branch of the function includes $x=1$

(b) $\lim_{x \to 0} f(x) = 0$

(c) $\lim_{x \to 1} f(x) =$ DNE

$\lim_{x \to 1^-} f(x) = 0 \neq \lim_{x \to 1^+} f(x) = 2$

. Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \dfrac{t+6}{t^2 - t} & \text{for } -1 < t < 2 \\ 3t - 2 & \text{for } t \geq 2 \end{cases}$$

(a) $f(-3/2) =$ DNE; the function is not defined between -1 and -2

(b) $f(2) = 4$ b/c $3(2)-2=4$

(c) $f(3/2) = 10$ use middle branch

(d) $\lim_{t \to -2} f(t) =$ DNE b/c right hand limit DNE

(e) $\lim_{t \to -1^+} f(t) = \frac{5}{2}$

(f) $\lim_{t \to 2} f(t) = 4$

(g) $\lim_{t \to 0} f(t) =$ DNE

use middle branch

$\lim_{t \to 0} \frac{t+6}{t^2 - t} = \lim_{t \to 0} \frac{t+6}{t(t-1)}$ Doesn't fix anything! Nothing can be simplified.

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \to d^-} f(x)$, (b) $\lim_{x \to d^+} f(x)$, and (c) $\lim_{x \to d} f(x)$ for the given function f and number d.

47)

$$f(x) = \begin{cases} x^2 - 2, & \text{for } x < 0, \\ 1, & \text{for } x \geq 0 \end{cases}$$

d = −4 → only in top branch

$\lim_{x \to -4^-} f(x) = 14$

$\lim_{x \to -4^+} = 14$

$\lim_{x \to -4} f(x) = 14$

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$$f(x) = \begin{cases} -4x - 3, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -2x - 3, & \text{for } x > 1 \end{cases}$$

don't care at $x=1$

d = 1

top branch

$\lim_{x \to 1^-} f(x) = -7$

$\lim_{x \to 1^+} f(x) = -5$

$\lim_{x \to 1} f(x) \Rightarrow$ DNE

3. Evaluate each limit using algebraic techniques.
Use ∞, $-\infty$ or DNE where appropriate.

(a) $\displaystyle \lim_{x \to 0} \frac{x^2 - 25}{x^2 - 4x - 5}$ $= 5$

(b) $\displaystyle \lim_{x \to 5} \frac{x^2 - 25}{x^2 - 4x - 5}$ $= \frac{10}{6} = \frac{5}{3}$

(c) $\displaystyle \lim_{x \to 1} \frac{7x^2 - 4x - 3}{3x^2 - 4x + 1}$ $= 5$

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(d) $\displaystyle \lim_{x \to -2} \frac{x^4 + 5x^3 + 6x^2}{x^2(x+1) - 4(x+1)}$ $= 1$

(e) $\displaystyle \lim_{x \to -3} |x + 1| + \frac{3}{x}$ $= 1$

(f) $\displaystyle \lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x^2 - 9}$ $= 1/24$

(g) $\displaystyle \lim_{x \to 3} \frac{\sqrt{x^2 + 7} - 3}{x + 3}$ $= \frac{1}{6}$

(h) $\displaystyle \lim_{x \to 2} \frac{x^2 + 2x - 8}{\sqrt{x^2 + 5} - (x+1)}$ $= -18$

(i) $\displaystyle \lim_{y \to 5} \left(\frac{2y^2 + 2y + 4}{6y - 3} \right)^{1/3}$ $= \frac{4}{3}$

(j) $\displaystyle \lim_{x \to 0} \sqrt[4]{2\cos(x) - 5}$ $\Rightarrow$ DNE

(k) $\displaystyle \lim_{x \to 0} \frac{\frac{1}{3+x} - \frac{1}{3-x}}{x}$ $= \frac{-2}{9}$

(l) $\displaystyle \lim_{x \to -6} \frac{\frac{2x+8}{x^2 - 12} - \frac{1}{x}}{x + 6}$ $= \frac{1}{36}$

a) $\lim_{x\to 0} \frac{x^2-25}{x^2-4x-5} = \frac{-25}{-5} = \boxed{5}$

b) $\lim_{x\to 5} \frac{x^2-25}{x^2-4x-5} = \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+1)} = \frac{10}{6} = \boxed{\frac{5}{3}}$

c) $\lim_{x\to 1} \frac{7x^2-4x-3}{3x^2-4x+1} = \frac{\cancel{(x-1)}(7x+3)}{\cancel{(x-1)}(3x-1)} = \frac{10}{2} = \boxed{5}$

d) $\lim_{x\to -2} \frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)} = \frac{x^2(x^2+5x+6)}{x^3+x^2-4x-4} = \frac{x^2(x+3)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}(x-2)}$

$\lim_{x\to -2} \frac{x^2(x+3)}{(x+1)(x-2)} = \frac{4(1)}{(-1)(-4)} = \frac{4}{4} = \boxed{1}$

e) $\lim_{x\to -3} |x+1| + \frac{3}{x} = |-3+1| + \frac{3}{-3} = |-2| - 1 = \boxed{1}$

f) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)} = \frac{(x+1)-4}{(x^2-9)(\sqrt{x+1}+2)} = \frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+3)(\sqrt{x+1}+2)}$

$= \frac{1}{(x+3)(\sqrt{x+1}+2)} = \frac{1}{(6)(2+2)} = \boxed{\frac{1}{24}}$

g) $\lim_{x\to 3} \frac{\sqrt{x^2+7}-3}{x+3} = \frac{\sqrt{16}-3}{6} = \boxed{\frac{1}{6}}$

h) $\lim_{x\to 2} \frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)} \cdot \frac{\sqrt{x^2+5}+(x+1)}{\sqrt{x^2+5}+(x+1)} = \lim_{x\to 2} \frac{(x^2+2x-8)(\sqrt{x^2+5}+(x+1))}{x^2+5-(x+1)^2} =$

$\hookrightarrow x^2+5-(x^2+2x+1)$

$\lim_{x\to 2} \frac{(x^2+2x-8)(\sqrt{x^2+5}+(x+1))}{-2x+4} = \lim_{x\to 2} \frac{(x+4)\cancel{(x-2)}(\sqrt{x^2+5}+(x+1))}{-2\cancel{(x-2)}} = \lim_{x\to 2} \frac{(x+4)(\sqrt{x^2+5}+(x+1))}{-2}$

$= -18$

i) $\lim_{y\to 5}\left(\frac{2y^2+2y+4}{6y-3}\right)^{1/3} = \left(\frac{2(25)+10+4}{30-3}\right)^{1/3} = \sqrt[3]{\frac{64}{27}} = \boxed{\frac{4}{3}}$

j) $\lim_{x\to 0}\sqrt[4]{2\cos(x)-5} \Rightarrow$ DNE $\sqrt[4]{2\cos(x)-5}$ does not exist near $x=0$

K) $\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3-x}}{x} = \lim_{x\to 0}\frac{\frac{3-x}{(3+x)(3-x)}-\frac{3+x}{(3+x)(3-x)}}{x} = \lim_{x\to 0}\frac{\frac{(3-x)-(3+x)}{(3+x)(3-x)}}{x}$

$= \lim_{x\to 0}\frac{\frac{-2x}{(3+x)(3-x)}}{x} = \lim_{x\to 0}\frac{-2}{(3+x)(3-x)} = \boxed{\frac{-2}{9}}$

l) $\lim_{x\to -6}\frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6} = \lim_{x\to -6}\frac{\frac{x(2x+8)}{x(x^2-12)}-\frac{x^2-12}{x(x^2-12)}}{x+6} = \lim_{x\to -6}\frac{\frac{x(2x+8)-(x^2-12)}{x(x^2-12)}}{x+6}$

$\lim_{x\to -6}\frac{\frac{2x^2+8x-x^2+12}{x(x^2-12)}}{x+6} = \lim_{x\to -6}\frac{x^2+8x+12}{x(x^2-12)(x+6)} = \lim_{x\to -6}\frac{\cancel{(x+6)}(x+2)}{x(x^2-12)\cancel{(x+6)}} = \lim_{x\to -6}\frac{x+2}{x(x^2-12)}$

$= \frac{-4}{-144} = \boxed{\frac{1}{36}}$

There are lots of approaches to evaluating limits so we need to be REALLY flexible. You need to be able to think through many different strategies.

More Difficult Analytical Limit Examples:

4a. $\lim\limits_{x\to 0} \frac{\sin(4x)}{x}$

$\lim\limits_{x\to 0} \frac{4\sin(4x)}{4x} = 4 \cdot 1 = \boxed{4}$ $\to 1$

b. $\lim\limits_{x\to 0} \frac{\cos(2x)}{x} \Rightarrow \boxed{DNE}$

$\lim\limits_{x\to 0^-} \frac{\cos(2x)}{x} \Rightarrow \frac{1}{-\text{close to } 0} = -\infty$

$\lim\limits_{x\to 0^+} \frac{\cos(2x)}{x} \Rightarrow \frac{1}{+\text{close to } 0} = \infty$

c. $\lim\limits_{x\to 0} \frac{\sin(x)}{2x^2-x} = \lim\limits_{x\to 0} \left[\frac{\sin(x)}{x} \cdot \frac{1}{2x-1}\right]$ $\to 1$

$= 1(-1) = \boxed{-1}$

d. $\lim\limits_{x\to 0} \frac{2x+\sin(x)}{x}$

$\lim\limits_{x\to 0} \frac{2x}{x} + \frac{\sin(x)}{x} =$

$\lim\limits_{x\to 0} 2 + \frac{\sin(x)}{x} = \boxed{3}$

e. $\lim\limits_{x\to 3} \frac{x-3}{\sqrt{4x+4}-4} \cdot \frac{\sqrt{4x+4}+4}{\sqrt{4x+4}+4}$

$\lim\limits_{x\to 3} \frac{(x-3)(\sqrt{4x+4}+4)}{4x+4-16}$

$\lim\limits_{x\to 3} \frac{(x-3)(\sqrt{4x+4}+4)}{4x-12}$

$\lim\limits_{x\to 3} \frac{\cancel{(x-3)}(\sqrt{4x+4}+4)}{4\cancel{(x-3)}} = \boxed{2}$

f. $\lim\limits_{x\to 2} \frac{\sqrt{3+x}-2}{x-1} = \frac{\sqrt{5}-2}{1}$

$= \boxed{\sqrt{5}-2}$

g. $\lim\limits_{x\to 0} \tan(x)$

$\lim\limits_{x\to 0} \frac{\sin(x)}{\cos(x)} = \boxed{0}$

h. If $j(x) = x^4 - 16$ and $h(x) = \frac{j(x)}{x+2}$, find $\lim\limits_{x\to -2} h(x)$.

$\lim\limits_{x\to -2} \frac{x^4-16}{x+2} = \lim\limits_{x\to -2} \frac{(x^2-4)(x^2+4)}{x+2}$

$= \lim\limits_{x\to -2} \frac{(x-2)\cancel{(x+2)}(x^2+4)}{\cancel{x+2}} = \lim\limits_{x\to -2} (x-2)(x^2+4) = -4(8)$

$= \boxed{-32}$