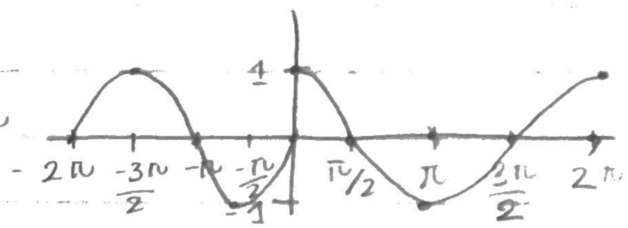
Hw 1.4

pg 68; 61, 63 65, 68
Pg 76; 9-16, 2[illegible], [illegible], 31, 33
Pg 77; Quick Quiz 1, 2

Pg 68 61)

a) $f(x) = \begin{cases} \sin x, & -2\pi \leq x < 0 \\ \cos x, & 0 \leq x \leq 2\pi \end{cases}$



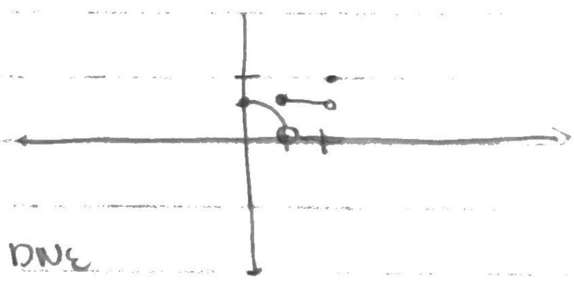
b) $\lim_{x\to0^-} f(x) \neq \lim_{x\to0^+} f(x) \therefore \lim_{x\to0} f(x) = DNE$. lim exist everywhere else on $(-2\pi, 2\pi)$

c) The left hand only exist at 2π

d) The right hand only exist at -2π

63) $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2. & x = 2 \end{cases}$

a)



b) $\lim_{x\to1^-} f(x) \neq \lim_{x\to1^+} f(x) \therefore \lim_{x\to1} f(x) = DNE$

$\lim_{x\to2^-} f(x) \neq \lim_{x\to2^+} f(x) \therefore \lim_{x\to2} f(x) = DNE$

Lim exist everywhere else on $(0,1)$ and $(1,2)$

c) The left hand only exist at $x=2$

d) The right hand only exist at $x=0$

65) $\lim_{x\to0} x\sin x$

$-1 \leq \sin x \leq 1$

$-x \leq x\sin x \leq x$

$\lim_{x\to0} -x \leq \lim_{x\to0} x\sin x \leq \lim_{x\to0} x \rightarrow 0 \leq \lim_{x\to0} x\sin x \leq 0$

$\therefore \lim_{x\to0} x\sin x = 0$

66) $\lim_{x\to 0} x^2 \sin x$

$$-1 \le \sin x \le 1$$
$$-x^2 \le x^2 \sin x \le x^2$$
$$\lim_{x\to 0} -x^2 \le \lim_{x\to 0} x^2 \sin x \le \lim_{x\to 0} x^2$$
$$0 \le \lim_{x\to 0} x^2 \sin x \le 0 \quad \therefore \lim_{x\to 0} x^2 \sin x = 0$$

67) $\lim_{x\to 0} x^2 \sin(1/x^2)$

$$-1 \le \sin(1/x^2) \le 1$$
$$-x^2 \le x^2 \sin(1/x^2) \le x^2$$
$$\lim_{x\to 0} -x^2 \le \lim_{x\to 0} x^2 \sin(1/x^2) \le \lim_{x\to 0} x^2$$
$$0 \le \lim_{x\to 0} x^2 \sin(1/x^2) \le 0$$
$$\therefore \lim_{x\to 0} x^2 \sin(1/x^2) = 0$$

68) $\lim_{x\to 0} x^2 \cos(1/x^2)$

$$-1 \le \cos(1/x^2) \le 1$$
$$-x^2 \le x^2 \cos(1/x^2) \le x^2$$
$$\lim_{x\to 0} -x^2 \le \lim_{x\to 0} x^2 \cos(1/x^2) \le \lim_{x\to 0} x^2$$
$$0 \le \lim_{x\to 0} x^2 \cos(1/x^2) \le 0$$
$$\therefore \lim_{x\to 0} x^2 \cos(1/x^2) = 0$$

pg 76 a) $\lim_{x\to \infty} \frac{1-\cos x}{x^2}$

$$0 \le 1-\cos x \le 2$$
$$0/x^2 \le (1-\cos x)/x^2 \le 2/x^2$$
$$\lim_{x\to \infty} 0/x^2 \le \lim_{x\to \infty} (1-\cos x)/x^2 \le \lim_{x\to \infty} 2/x^2$$
$$0 \le \lim_{x\to \infty} (1-\cos x)/x^2 \le 0$$
$$\therefore \lim_{x\to \infty} (1-\cos x)/x^2 = 0$$

(same as #9)

10) $\lim_{x\to-\infty} (1-\cos x)/x^2$ $\quad 0 \le (1-\cos x) \le 2$

$$0/x^2 \le (1-\cos x)/x^2 \le 2/x^2$$

$$\lim_{x\to\infty} 0/x^2 \le \lim_{x\to\infty} (1-\cos x)/x^2 \le \lim_{x\to-\infty} 2/x^2$$

$$0 \le \lim_{x\to-\infty} (1-\cos x)/x^2 \le 0$$

$$\therefore \lim_{x\to-\infty} (1-\cos x)/x^2 = 0$$

11) $\lim_{x\to\infty} \sin x/x$: $\quad -1 \le \sin x \le 1$

$$-1/x \le \sin x/x \le 1/x$$

$$\lim_{x\to-\infty} -1/x \le \lim_{x\to-\infty} \sin x/x \le \lim_{x\to-\infty} 1/x$$

$$0 \le \lim_{x\to-\infty} \sin x/x \le 0 \quad \therefore \lim_{x\to-\infty} \sin x/x = 0$$

12 $\lim_{x\to\infty} \frac{\sin(x^2)}{x}$ $\quad -1 \le \sin x^2 \le 1$

$$-1/x \le ((\sin x^2)/x) \le 1/x$$

$$\lim_{x\to\infty} -1/x \le \lim_{x\to\infty} (\sin x^2)/x \le \lim_{x\to\infty} 1/x$$

$$0 \le \lim_{x\to\infty} (\sin x^2)/x \le 0$$

$$\therefore \lim_{x\to\infty} (\sin x^2)/x = 0$$

Note, 13-16 (you do not have to make tables/graphs)

13) $\lim_{x\to2^+} \frac{1}{x-2} \rightarrow \lim_{x\to2^+} \frac{1}{\text{positive close to 0}} = \infty$

14) $\lim_{x\to2^-} x/x-2 \rightarrow \lim_{x\to2^-} \frac{2}{\text{negative close to 0}} = -\infty$

15) $\lim_{x\to-3^-} \frac{1}{x+3} \rightarrow \lim_{x\to-3^-} = \frac{1}{\text{-close to 0}} = -\infty$

16) $\lim_{x \to -3^+} \frac{x}{x+3} \to \lim_{x \to -3^+} \frac{-3}{+\text{close to } 0} = -\infty$

27) $f(x) = \frac{1}{x^2-4}$ $\quad x^2-4=0$

a) $(x-2)(x+2)$ $\quad$ VA: $x=2$, $x=-2$

b) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{1}{(-)(+)} = -\infty$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{1}{(+)(+)} = +\infty$

$\lim_{x \to -2^-} f(x) = \lim_{x \to 2^-} \frac{1}{(-)(-)} = +\infty$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{1}{(-)(+)} = -\infty$

29) $f(x) = \frac{x^2-2x}{x+1}$ $\quad (x+1)=0$ $\quad$ VA: $x=-1$

31,33 a) $\quad x=-1$

b) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x(x-2)}{x+1} = \lim_{x \to -1^-} = \frac{-1(-1-2)}{-\text{close to } 0} = \frac{+}{-} = -\infty$

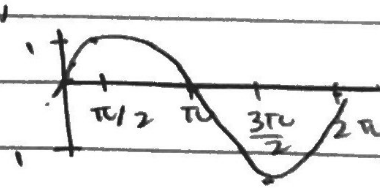
$\lim_{x \to -1^+} f(x) = \frac{+}{+\text{close to } 0} = \infty$

31) $f(x) = \cot x = \frac{\cos x}{\sin x}$ $\quad \sin x = 0$

$x = 0, \pi,$

Note interval $0 < x < 2\pi$

$\lim_{x \to 0^+} f(x) = \frac{+}{+\text{close to } 0} = +\infty$



$\lim_{x \to 0^-} f(x) = \frac{+}{-\text{close to } 0} = -\infty$

$\lim_{x \to \pi^+} f(x) = \frac{-}{-\text{close to } 0} = +\infty$

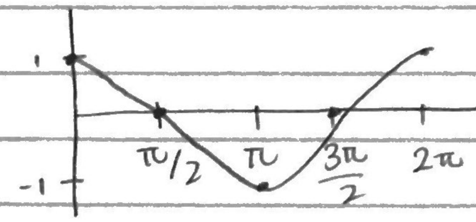
$\lim_{x \to \pi^-} f(x) = \frac{-}{+\text{close to } 0} = -\infty$

33) $f(x) = \tan x/\sin x = \frac{\cancel{\sin x}}{\cos x} \cdot 1/\cancel{\sin x} = 1/\cos x = \sec x$

VA $= \pi/2,\ 3\pi/2$ $\qquad\qquad$ $\cos x = 0$

$\lim\limits_{x \to \pi/2^+} f(x) = \frac{1}{-\text{close to } 0} = -\infty$ $\qquad$ $x = \pi/2,\ 3\pi/2$ on $(0, 2\pi)$

$\lim\limits_{x \to \pi/2^-} f(x) = \frac{1}{+\text{close to } 0} = +\infty$



$\lim\limits_{x \to 3\pi/2^+} f(x) = \frac{1}{+\text{close to } 0} = +\infty$

$\lim\limits_{x \to 3\pi/2^-} f(x) = \frac{1}{-\text{close to } 0} = -\infty$

Pg 77 Quick Quiz

#1 $\lim\limits_{x \to 3} \frac{x^2 - x - 6}{x - 3}$ $\quad$ $\lim\limits_{x \to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}} = (3+2) = 5$ $\boxed{D}$

#2 Find $\lim\limits_{x \to 2^+} f(x)$

$f(x) = \begin{cases} 3x+1, & x < 2 \\ 5/x+1, & x \geq 2 \end{cases}$ $\qquad$ $5/(x+1)$

$5/(2+1) = 5/3$

$\lim\limits_{x \to 2^+} f(x) = 5/3$, A