HW 1.5

Pg. 84: 1-27 (odd), 41, 43, 47, 49

1. $y = \frac{1}{(x+2)^2}$ Continuous everywhere except $x=-2$; infinite discontinuity because a Vertical Asymptote is at $x=-2$.

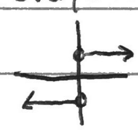
3. $y = \frac{1}{x^2+1}$ continuous everywhere; no discontinuities (nothing makes denominator $=0$)

5. $y = \sqrt{2x+3}$ continuous if $x \geq -\frac{3}{2}$, function is discontinuous if $x < \frac{-3}{2}$ because those points (x-values) are not in the domain.

$2x+3 \geq 0$

$x \geq -\frac{3}{2}$

7. $y = \frac{|x|}{x}$ continuous everywhere except $x=0$; jump discontinuity at $x=0$



9. $y = e^{1/x}$ continuous everywhere except $x=0$; infinite discontinuity at $x=0$ because as $x \to 0^+$, $\frac{1}{x} \to \infty$, so $e^{\frac{1}{x}} \to \infty$

11. a) yes $f(-1)=0$

b) $\lim_{x \to -1^+} f(x) = 0$ yes

c) $\lim_{x \to -1^+} f(x) = f(-1) = 0$ yes

d) yes. This is an interesting case; $x=-1$ is the end of the domain, so we do not care about $\lim_{x \to -1^-} f(x)$

13. No, $f(x)$ is not defined at $x=2$. There is a hole

15. $f(2)=0$ would make $f(x)$ continuous at $x=2$.

17. No; the $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+}$ so there is a jump discontinuity. That can't be "filled"

19. a) Discontinuity at $x=2$ because $f(x)$ is not defined when $x=2$

b) $\lim_{x\to 2^-} 3-x = 1$

$\lim_{x\to 2^+} \frac{x}{2}+1 = 2$

This means there is a jump discontinuity at $x=2$ b/c $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$. The discontinuity at $x=2$ is not removable.

21. a) Discontinuity at $x=1$

b) $\lim_{x\to 1^-} f(x) \to -\infty$

$\lim_{x\to 1^+} f(x) = 4$

This means there is an infinite discontinuity. Therefore the discontinuity at $x=1$ is not removable.

23. a) Discontinuity at $x=0$, $x=1$, and all points not in the domain

b) The discontinuity at $x=0$ is removable b/c $f(0)$ can be redefined as $f(0)=0$ and "plug the hole." The discontinuity at $x=1$ is not removable b/c there is a jump discontinuity. $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

25. $f(x) = \frac{x^2-9}{x+3}$ $\lim_{x\to -3} \frac{x^2-9}{x+3} = \lim_{x\to -3} \frac{(x-3)(x+3)}{(x+3)} = \lim_{x\to -3} x-3 = -6$

$$f(x) = \begin{cases} \frac{x^2-9}{x+3}, & x \neq -3 \\ -6, & x=-3 \end{cases}$$

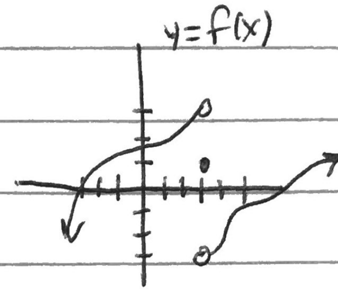
or simply say $f(x) = x-3$

27. $f(x) = \frac{\sin(x)}{x}$ $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

$$f(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0 \\ 1, & x=0 \end{cases}$$

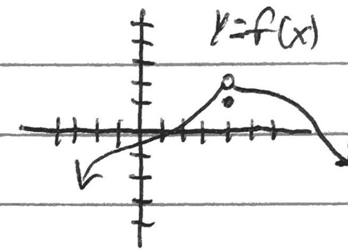
41. $f(3)$ exists but $\lim\limits_{x\to 3} f(x)$ does not.

Graphs will vary



43. $f(4)$ exists, $\lim\limits_{x\to 4} f(x)$ exists, but f is not continuous at $x=4$.

Graphs will vary



47. $f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\geq 3 \end{cases}$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$$

$f(3)=6a$; so point exists

$$x^2-1 = 2ax \quad \text{at } x=3$$

$$8 = 6a$$

$$\frac{8}{6} = a$$

$$a = \frac{4}{3}$$

49. $f(x)=\begin{cases} 4-x^2, & x<-1 \\ ax^2-1, & x\geq -1 \end{cases}$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

$f(-1)=a-1$; so point exists.

$$4-x^2 = ax^2-1 \quad \text{at } x=1$$

$$3 = a-1$$

$$4=a$$