Section 1: Algebra Review

1. Solve $xy + 2x + 1 = y$ *for* y

$$xy - y = -2x - 1$$
$$y(x-1) = -2x - 1$$
$$y = \frac{-2x-1}{x-1}$$

2. Factor: $x^2(x-1) - 4(x-1)$

$$(x-1)[x^2 - 4]$$
$$(x-1)(x-2)(x+2)$$

3. Solve $\ln(y-1) - \ln x = x + \ln x$ *for* y

$$e^{\ln(y-1)} = e^{x + 2\ln(x)}$$
$$y - 1 = e^{x+2\ln(x)}$$
$$y = e^{x+2\ln(x)} + 1$$

4. Factor $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

$$3x^{-\frac{1}{2}}[x^{\frac{4}{2}} - 3x^{\frac{2}{2}} + 2]$$
$$3x^{-\frac{1}{2}}[x^2 - 3x + 2]$$
$$3x^{-\frac{1}{2}}[x-2][x-1]$$

Simplify each expression.

5. $\frac{(x^2)^3 x}{x^7}$

$$\frac{x^6 x}{x^7} = \frac{x^7}{x^7} = 1$$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{6}}$$
$$x^{\frac{3}{6}} \cdot x^{\frac{2}{6}} \cdot x^{\frac{1}{6}}$$
$$x^{\frac{6}{6}} = x$$

7. $\frac{5(x+h)^2 - 5x^2}{h}$

$$\frac{5(x^2 + 2xh + h^2) - 5x^2}{h}$$
$$\frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$
$$\frac{10xh + 5h^2}{h}$$
$$10x + 5h$$

This is derivative! (but w/o the $\lim_{h \to 0}$)

8. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

$$\frac{\frac{x}{x^2} + \frac{4}{x^2}}{\frac{3x}{x} - \frac{1}{x}}$$
$$\frac{\frac{x+4}{x^2}}{\frac{3x-1}{x}}$$
$$\frac{(x+4)x}{x^2(3x-1)}$$
$$\frac{x+4}{x(3x-1)}$$

Simplify, by factoring first. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-5x-9-16x-9]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-21x-18]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

9. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$

$(x-1)^2[(x-1)(2x-3)-(2x+12)]$

$(x-1)^2[2x^2-3x-2x+3-2x-12]$

$(x-1)^2[2x^2-7x-9]$

$(x-1)^2(2x-9)(x+1)$

10. $\frac{(x-1)^2(3x-1)-2(x-1)}{(x-1)^4}$

$\frac{(x-1)[(x-1)(3x-1)-2]}{(x-1)^4}$

$\frac{(x-1)[3x^2-x-3x+1-2]}{(x-1)^4}$

$\frac{(x-1)[3x^2-4x-1]}{(x-1)^4} = \frac{3x^2-4x-1}{(x-1)^3}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \bullet \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

These are derivatives w/ limits!

11. $\frac{\sqrt{x+9}-3}{x} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}$

$\frac{(x+9)-9}{x(\sqrt{x+9}+3)} = \frac{x}{x(\sqrt{x+9}+3)} = \frac{1}{\sqrt{x+9}+3}$

12. $\frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^4 - 2x^2 = 0$

$5x^4 - 2x^2 = 0$

$x^2[5x^2 - 2] = 0$

$x^2(\sqrt{5}x - \sqrt{2})(\sqrt{5}x + \sqrt{2}) = 0$

$x = 0$

$x = \frac{\sqrt{2}}{\sqrt{5}}$ or $\frac{\sqrt{10}}{5}$

$x = -\frac{\sqrt{2}}{\sqrt{5}}$ or $\frac{-\sqrt{10}}{5}$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

$(2x-7)(x+4) = (x+1)(2x)$

$2x^2 + 8x - 7x - 28 = 2x^2 + 2x$

$8x - 7x - 28 = 2x$

$-x - 28 = 0$

$-28 = x$

15. $\sqrt{x^2 - 9} = x - 1$

$x^2 - 9 = (x-1)^2$

$x^2 - 9 = x^2 - 2x + 1$

$-9 = -2x + 1$

$-10 = -2x$

$x = 5$

16. $|2x - 3| = 14$

$2x - 3 = 14$ OR $2x - 3 = -14$

$2x = 17$ $\quad$ $2x = -11$

$x = \frac{17}{2}$ $\quad$ $x = -\frac{11}{2}$

17. $x^2 - 2x - 8 < 0$ (answer in interval notation)

$(x-4)(x+2) < 0$

		-2		4	
	$+$		$-$		$+$
$x-4$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$

$(-2, 4)$

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

$3x + 5 = 0$

$x = -\frac{5}{3}$

Solve each system of equations algebraically and graphically.

19. $\begin{cases} x + y = 8 \\ 2x - y = 7 \end{cases}$ +

$3x = 15$

$x = 5$

$5 + y = 8$

$y = 3$ $(5,3)$

Graph to check intersection.

20. $\begin{cases} y = x^2 - 3x \\ y = 2x - 6 \end{cases}$

$x^2 - 3x = 2x - 6$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ and $x = 2$

$y = 2(3) - 6$ $y = 2(2) - 6$

$y = 0$ $y = -2$

$(3,0)$ $(2,-2)$

Graph to check intersections

Section 2: Trigonometry Review

21. Use your knowledge of the unit circle, to evaluate each of the following. You MUST know your unit circle. Leave answers in radical form. Do NOT use your calculator.

a) $sin30° = \frac{1}{2}$

b) $cos\frac{2\pi}{3} = -\frac{1}{2}$

c) $tan45° = \frac{\sqrt{2}}{2}$

d) $sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

e) $tan\pi = 0$

f) $cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

g) $\cos(90°) = 0$

h) $cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

i) $cot\frac{\pi}{6} = \sqrt{3}$

j) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

k) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

l) $\tan^{-1}(1) = \frac{\pi}{4}$

Solve each trigonometric equation for $0 \le x \le 2\pi$.

22. $sinx = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

23. $\tan^2 x = 1$

$\tan(x) = \pm 1$

$\tan^{-1}(x) = 1 \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan^{-1}(x) = -1 \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$

24. $cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ on $0 \to 2\pi$

$\frac{x}{2} = \frac{\pi}{4}$ or $\frac{x}{2} = \frac{7\pi}{4}$

$x = \frac{\pi}{2}$ or ~~$x = \frac{7\pi}{2}$~~

Not in domain

25. $2\sin^2 x + sinx - 1 = 0$

$(2\sin(x) - 1)(\sin(x) + 1) = 0$

$2\sin(x) - 1 = 0 \qquad \sin(x) + 1 = 0$

$\sin(x) = \frac{1}{2} \qquad \sin(x) = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$

26. $3cosx + 3 = 2\sin^2 x$

$3\cos(x) + 3 = 2(1 - \cos^2(x))$

$3\cos(x) + 3 = 2 - 2\cos^2(x)$

$2\cos^2(x) + 3\cos(x) + 1 = 0$

$(2\cos(x) + 1)(\cos(x) + 1) = 0$

$2\cos(x) + 1 = 0 \qquad \cos(x) + 1 = 0$

$\cos(x) = -\frac{1}{2} \qquad \cos(x) = -1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3} \qquad x = \pi$

Solve each exponential or logarithmic equation.

27. $5^x = 125$

$5^x = 5^3$

$x = 3$

28. $8^{x+1} = 16^x$

$8^{x+1} = 2^{4x}$

$(2^3)^{x+1} = 2^{4x}$

$2^{3x+3} = 2^{4x}$

$3x + 3 = 4x \quad x = 3$

29. $81^{\frac{3}{4}} = x$

$\sqrt[4]{81}^{\,3} = x$

$(3)^3 = x$

$27 = x$

30. $8^{-\frac{2}{3}} = x$

$\frac{1}{\sqrt[3]{8}^{\,2}} = x$

$\frac{1}{4} = x$

31. $\log_2 32 = x$

$5 = x$

32. $\log_x \frac{1}{9} = -2$

$x^{-2} = \frac{1}{9}$

$\frac{1}{x^2} = \frac{1}{9}$

$9 = x^2$

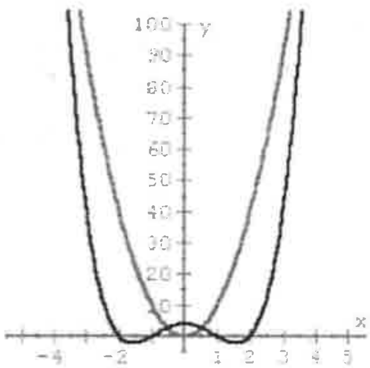
$x = \pm 3$ but in logs base is + $x = 3$

Section 5: Polynomial Functions

Polynomials : $f(x) = a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0$

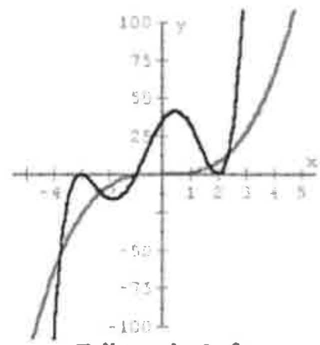
Studying End Behavior

Even Degree

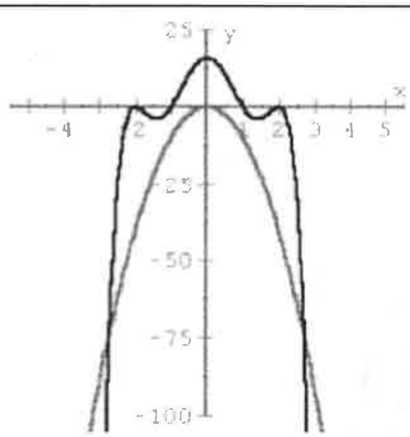


Rises on the Left
Rises on the Right
$f(x) = x^4 - 5x^2 + 4$
$a_n > 0$

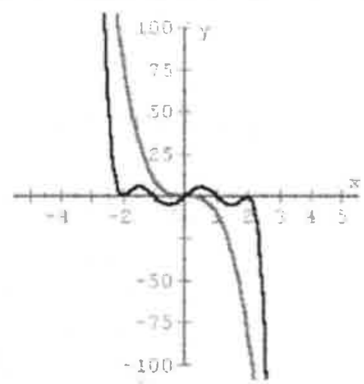
Odd Degree



Falls on the Left
Rises on the Right
$f(x) = x^5 + 3x^4 - 9x^3 - 23x^2 + 24x + 36$
$a_n > 0$



Falls on the Left
Falls on the Right
$f(x) = -x^6 + 9x^4 - 24x^2 + 16$
$a_n < 0$



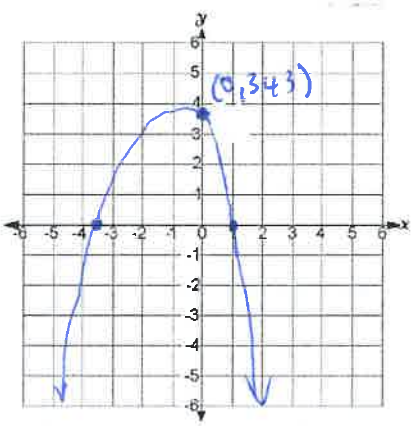
Rises on the Left
Falls on the Right
$f(x) = -x^7 + 9x^5 - 24x^3 + 16x$
$a_n < 0$

64. Sketch a graph of the function without using a calculator. Identify the y-intercept $X=0$ (although it will probably not have drawn to scale on the given grid).

a) $f(x) = -(2x+7)^3(x-1)$

$f(0) = -(7)^3(-1) = 7^3 = 343$

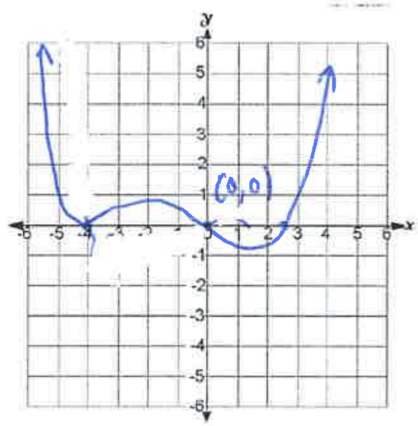
zeros: $x = \frac{-7}{2}, 1$



b) $g(x) = x^3(x+4)^2(2x-5)$

zeros: $X = 0, -4, \frac{5}{2}$

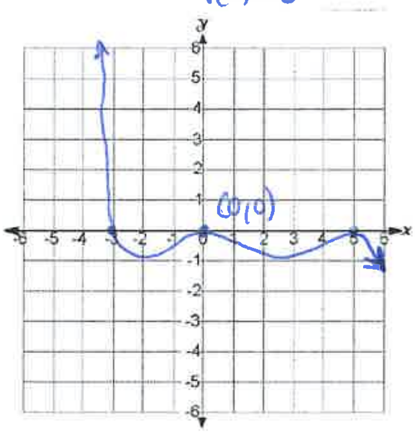
$g(0) = 0$



c) $f(x) = -x^2(x-5)^2(x+3)$

zeroes: $X = 0, 5, -3$

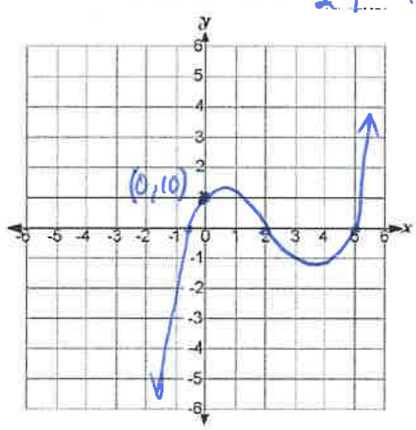
$f(0) = 0$



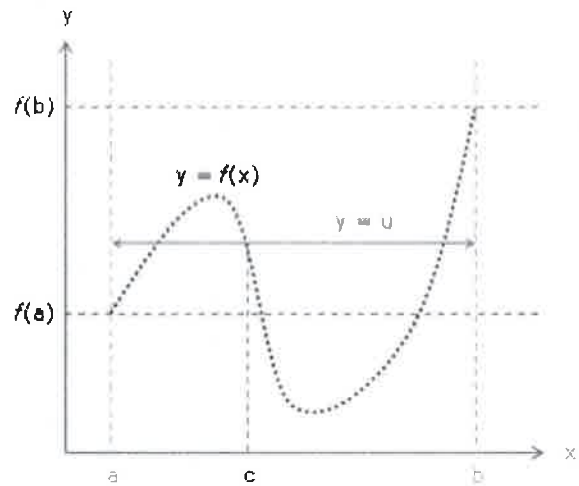
d) $f(x) = (2x+1)^3(x-2)(x-5)$

$f(0) = (1)(-2)(-5) = 10$

zeroes: $x = -\frac{1}{2}, 2, 5$



The **intermediate value theorem** (IVT) states the following: If the function $y = f(x)$ is continuous on the interval [a, b], and u is a number between $f(a)$ and $f(b)$, then there is a $c \in [a, b]$ such that $f(c) = u$.



Example: Suppose that we want to know if $f(x) = x^4 - 7x^3 - 4x + 8$ is ever zero.

Solution: Since this function is a polynomial, we know that it is continuous everywhere. At x = -1, we get $f(-1)=20$. At x = 1, we get $f(1) = -2$. So at the two endpoints of the interval [-1, 1], the functions has values of 20 and -2. Therefore, $f(x)$ must take on all values between -2 and 20 as x varies between - 1 and 1. In particular, $f(x)$ must take on the value 0 for some x in [-1, 1]. The Intermediate Value Theorem (IVT) does not tell us exactly **where** $f(x)$ equals 0, only that it **is** 0 somewhere on the interval [-1, 1].

65. Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere between -1 and 2.

$P(-1)=8$

$P(2)=-19$

P(x) is is continuous between x=-1 and x=2 b/c p(x) is a polynomial. P(-1)=8 and P(2)=-19; therefore p(x)=0 somewhere between x=-1 an 2.

66. Use the Intermediate Value Theorem to prove that the equation $x^3 = x + 8$ has at least one solution.

$x^3 = x+8$

$x^3 - x - 8 = 0$

$f(x) = x^3 - x - 8$

$f(4) = 52$

$f(1) = -8$

f(x) is continuous between x=1 and x=4 b/c f(x) is a polynomial. f(1)=-8 and f(4)=52; therefore f(x)=0 somewhere between x=1 and x=4. Thus, $x^3=x+8$ has at least one solution.

Section 6: Average Rate of Change Review

> Definition: Average Speed
> Average speed is found by dividing the distance covered by the elapsed time.
> $$\frac{\Delta y}{\Delta t} = \frac{\text{total distance traveled}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}}$$

not technically equal

average speed   average velocity

67. Find the average speed of a car that has traveled 350 miles in 7 hours.

$$\frac{350\text{mi}}{7\text{ hr}} = 50\text{ mi/hr}$$

68. Suppose $f(1)=2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

$$\frac{f(5)-f(1)}{5-1} = 3 \Rightarrow \frac{x-2}{4} = 3 \Rightarrow x-2=12$$
$$x = 14$$

69. The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t) = 3t^2 + 1$.

a) Find the change in position between times t = 1 and t = 3.

$p(3)=28$ $28-4=24\text{m}$
$p(1)=4$

b) Find the average velocity of the object between times t = 1 and t = 4.

$$\frac{p(4)-p(1)}{4-1} = \frac{49-4}{3} = \frac{45}{3} = 15$$

c) Find the average velocity of the object between any time t and another time $t + \Delta t$.

$$\frac{p(t+\Delta t)-p(t)}{(t+\Delta t)-t} = \frac{(3(t+\Delta t)^2+1)-(3t^2+1)}{\Delta t} = \frac{3t^2+6t(\Delta t)+(\Delta t)^2+1-3t^2-1}{\Delta t}$$

$$= \frac{6t(\Delta t)+(\Delta t)^2}{\Delta t} = 6t+\Delta t$$

Derivative (w/o limit)

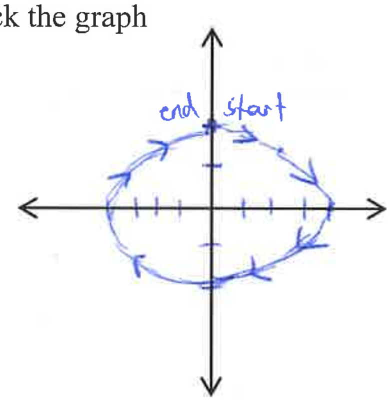
Section 7: Parametric Functions

Parametric equations are given below.

70. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Be sure to define the portion of the graph of the rectangular equation traced by the parametric equations.

a). $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$


check the graph



$\frac{x}{4} = \sin(t) \quad \frac{y}{2} = \cos(t)$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x	0	$2\sqrt{2}$	4	$2\sqrt{2}$	0	−4	0
y	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	2

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = \sin^2(t) + \cos^2(t)$

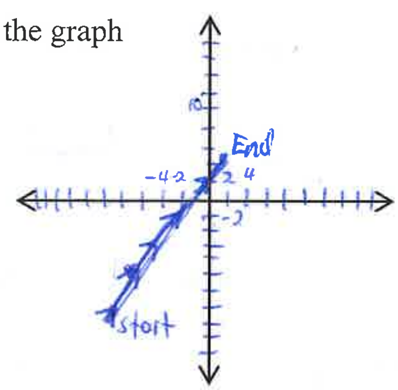
$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

pythagorean identity

one revolution of the ellipse was traced.

b) $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$


check the graph



$\frac{x+5}{2} = t$

t	−2	−1	0	1	2	3
x	−9	−7	−5	−3	−1	1
y	−15	−11	−7	−3	1	5

$Y = 4\left(\frac{x+5}{2}\right) - 7$

$Y = 2x + 10 - 7$

$Y = 2x + 3$ from $x_i(-9, 1)$

Section 8: Inverse Functions

71. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$

$x = \frac{3}{y-2} - 1$

$x+1 = \frac{3}{y-2}$

$(y-2)(x+1) = 3$

$y-2 = \frac{3}{x+1}$

$y = \frac{3}{x+1} + 2$

72. If $f(x) = x^3 - 1$, find f^{-1} and verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

$y = x^3 - 1$

$x = y^3 - 1$

$x+1 = y^3$

$\sqrt[3]{x+1} = y$

$f(f^{-1}(x)) = (\sqrt[3]{x+1})^3 - 1 = x + 1 - 1 = x$ ✓

$f^{-1}(f(x)) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$ ✓

73. Discuss the relationship between the domain and range of a function and its inverse.

In inverse functions, the domains and ranges are reversed. The domain and range of f(x) become the range and domain of $f^{-1}(x)$ respectively.

74. Given the one-to-one function *f*. The point *(a,c)* is on the graph of *f*. Give the coordinates of a point on the graph of f^{-1}.

(c, a)

75. Discuss the relationship between the graph of a function and the graph of its inverse. You can use an example to illustrate your answer.

The graph of f(x) is reflected over the line y=x to create the graph of $f^{-1}(x)$.